

Control – S-domain



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Laplace Transform

□ Laplace transform

- Laplace transform is named after Pierre-Simon Laplace, who introduced the transform in his work on probability theory.
- Laplace transform simplifies the process of analyzing the behavior of the system. In engineering applications, normally refer to s-domain, which corresponding to a linear time-invariant (LTI) system for system stability and dynamic analysis.
- Laplace transform definition

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

where $s = \sigma + j\omega$

- Reference:
http://en.wikipedia.org/wiki/Laplace_transform

Mathematical Relationship

Commonly Used in Electronics

$$f(t) \leftrightarrow F(s)$$

$$\frac{df(t)}{dt} \leftrightarrow sF(s)$$

$$\int f(t) dt \leftrightarrow \frac{1}{s} F(s)$$

Initial value theorem :

$$f(0^+) = \lim_{s \rightarrow \infty} sF(s)$$

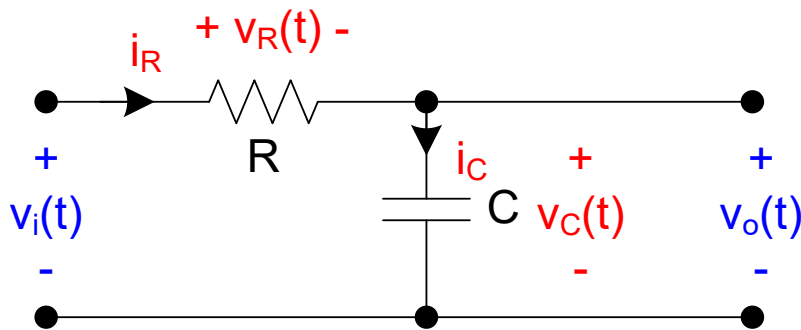
Final value theorem :

$$f(\infty) = \lim_{s \rightarrow 0} sF(s)$$

Example of Laplace Transform in electronics circuit

Example of RC Filter

Question: Calculation $V_o(s)/V_i(s)$



$$(1): i_R = i_C$$

$$(2): i_C = C \frac{dv_C(t)}{dt} = C \frac{dv_o(t)}{dt}$$

For

$$v_i(t) = v_R(t) + v_C(t)$$

$$v_i(t) = i_R R + v_o(t)$$

$$v_i(t) = i_C R + v_o(t)$$

$$v_i(t) = CR \frac{dv_o(t)}{dt} + v_o(t)$$

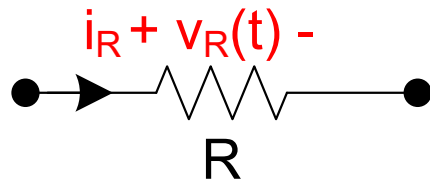
Laplace Transform

$$V_i(s) = sCRV_o(s) + V_o(s)$$

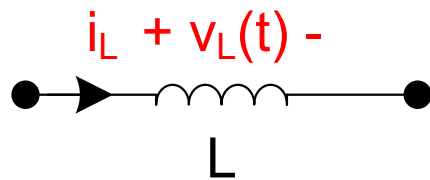
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sCR + 1}$$

Question: Do we need to setup differential equation first???

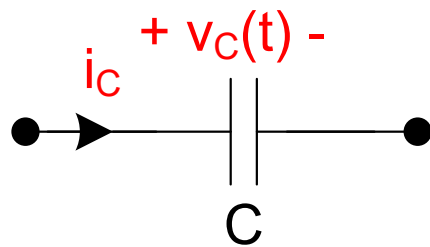
S-domain representation for circuit element



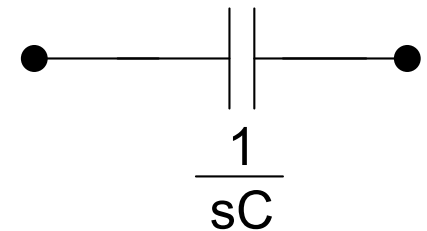
$$v_R(t) = Ri_R(t) \leftrightarrow V_R(s) = RI_R(s)$$
$$\therefore \frac{V_R(s)}{I_R(s)} = R$$



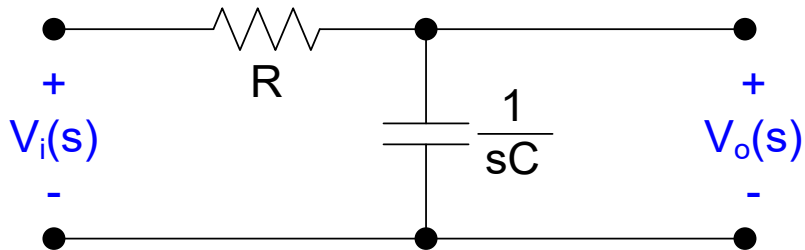
$$v_L(t) = L \frac{di_L(t)}{dt} \leftrightarrow V_L(s) = sLI_L(s)$$
$$\therefore \frac{V_L(s)}{I_L(s)} = sL$$



$$v_C(t) = \frac{1}{C} \int i_C(t) dt \leftrightarrow V_C(s) = \frac{1}{sC} I_C(s)$$
$$\therefore \frac{V_C(s)}{I_C(s)} = \frac{1}{sC}$$



Revisit RC example



$$V_o(s) = \frac{1}{sC} V_i(s) \frac{1}{R + \frac{1}{sC}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sCR}$$

Benefit: Simplify circuit analysis without differential equation!

What can we do with the s-domain transfer function

Example: RC Filter

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sCR + 1} \rightarrow V_o(s) = \frac{1}{sCR + 1} V_i(s)$$

If input is assumed to be unit step, i.e. $V_i(t) = \frac{1}{s}$

Apply Initial value theorem :

$$f(0^+) = \lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \frac{s}{sCR + 1} V_i(s)$$

$$f(0^+) = \lim_{s \rightarrow \infty} \frac{s}{sCR + 1} \frac{1}{s} = \lim_{s \rightarrow \infty} \frac{1}{sCR + 1} = 0$$

Apply Final value theorem

$$f(\infty) = \lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} \frac{s}{sCR + 1} V_i(s)$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{s}{sCR + 1} \frac{1}{s} = \lim_{s \rightarrow 0} \frac{1}{sCR + 1} = 1$$

Steady state output independent of C and R

Laplace transform can help to calculate the steady state response without solving complicated differential equation.

Input Test Signal and Corresponding Laplace s-domain

| Function | Time domain $f(t) = \mathcal{L}^{-1}\{F(s)\}$ | Laplace s-domain $F(s) = \mathcal{L}\{f(t)\}$ |
|-------------------|--|--|
| unit impulse | $\delta(t)$ | 1 |
| delayed impulse | $\delta(t - \tau)$ | $e^{-\tau s}$ |
| unit step | $u(t)$ | $\frac{1}{s}$ |
| delayed unit step | $u(t - \tau)$ | $\frac{e^{-\tau s}}{s}$ |
| ramp | $t \cdot u(t)$ | $\frac{1}{s^2}$ |

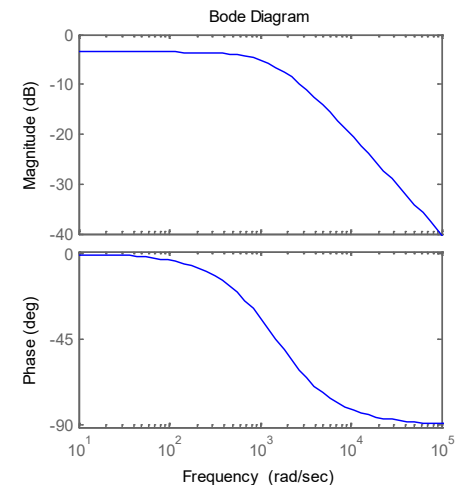
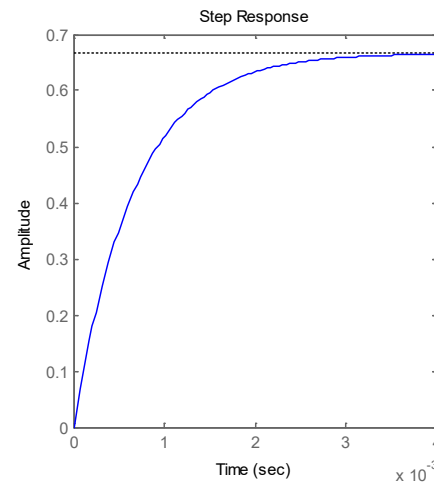
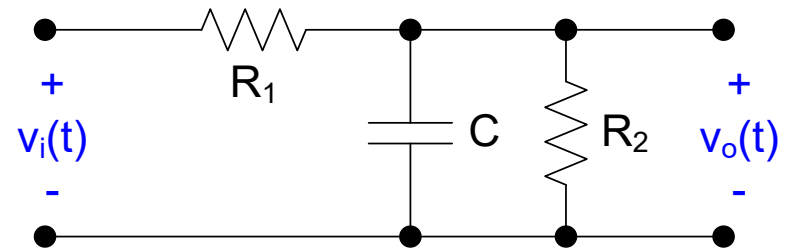
Benefit of s-domain

- In control theory, it well develop the understanding of 1st-order and 2st-order transfer function in s-domain. Therefore, without solving the equation, we can simply conclude the response of a system transfer function without solving the exact equation, and to design proper compensation network. This topic is discussed in another document
 - Control - System Response.doc
 - Control - Matlab and Control.doc

Homework – Question 1

Question 1

- Find system transfer function $G(s)=V_o(s)/V_i(s)$
- Use final value theorem to determine steady state value if unit step response is used.
- Assume $R_1=1k$, $R_2=2k$, $C=1\mu F$, use matlab to plot the step response and bode plot.
 - You will use matlab function
 - TF
 - STEP
 - BODE
 - Verify the step response with LTspice



Homework – Question 2

□ Question 2

- Calculate system transfer function $G_1(s)$ and $G_2(s)$.
- If $G_1(s)$ and $G_2(s)$ are connected in series (cascade), what is the new transfer function $G_{\text{overall}}(s)$?
- Explain why $G_{\text{overall}}(s)$ is not same as answer of question 1.

