Ch0 - Control for Power Electronics Engineering

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8-5-2021

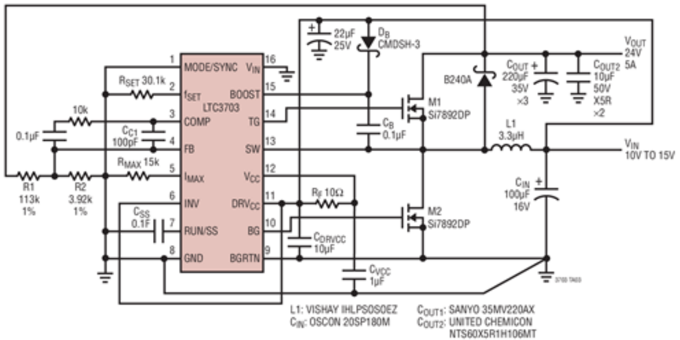
Control Theory – Definition in Wikipedia

- Control theory deals with the ***control of dynamical systems*** in engineered processes and machines. The ***objective*** is to develop a model or algorithm governing the application of system inputs ***to drive the system to a desired state***, while ***minimizing*** any ***delay, overshoot, or steady-state error*** and ensuring a level of ***control stability***; often with the aim to achieve a degree of optimality

From Physical Circuit to S-Domain

Physical World

Circuit Model



Laplace Transform
(S-Domain)

$$\mathscr{L}\{f(t)\} = F(s)$$

Ch1 : 1st and 2nd order Transfer Function

1st Order Transfer Function

$$G(s) = \frac{p}{s+p}$$

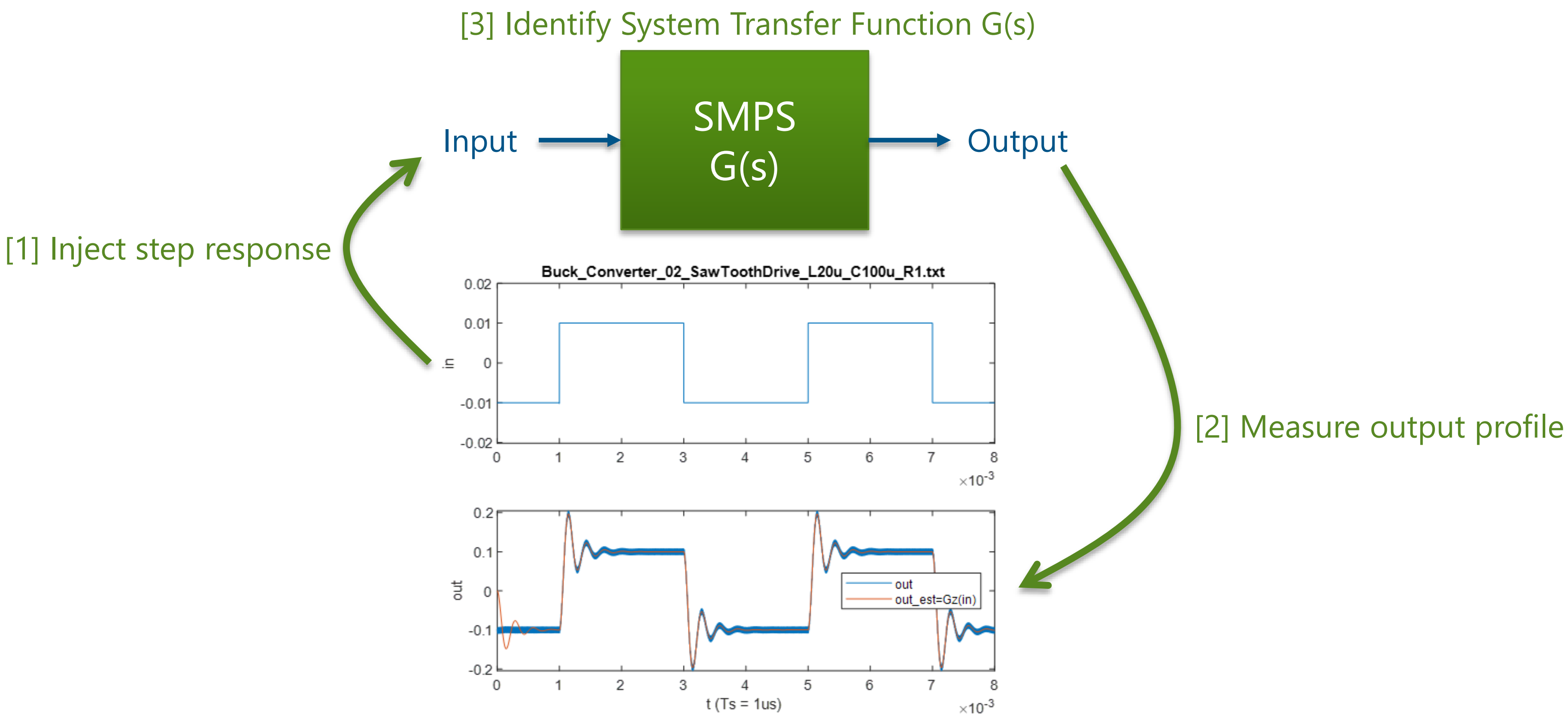
2nd Order Transfer Function

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

To Understand Basic System Dynamic

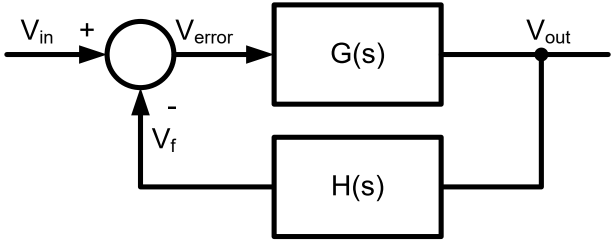
- Poles : Determine stability
  - LHP stable
  - RHP unstable
- Zeros : Affect transient dynamic
  - LHP increase overshoot
  - RHP step response starts in wrong direction
- Damping factor and natural frequency
  - Overshoot
  - System response time

Ch2 : System Identification Method



Ch3 : Close Loop and Open Loop System

Close-Loop Transfer Function

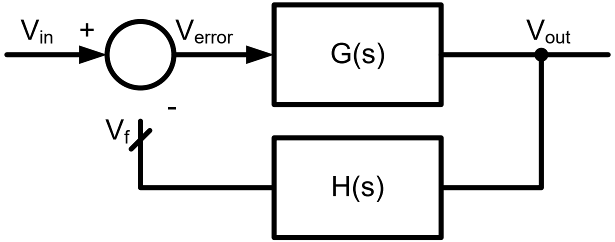


$$T(s) = \frac{V_{out}}{V_{in}} = \frac{G(s)}{1 + G(s)H(s)}$$

Goal is close-loop

But firstly back to something we familiar

Open-Loop Transfer Function



$$GH(s) = \frac{V_f}{V_{in}} = \frac{V_f}{V_{error}} = G(s)H(s)$$

Open Loop and Close Loop Relationship
Bode Method (Frequency Domain)

- DC Gain – Steady State Error
- Phase Margin – Damping Ratio
- Gain Margin – System Robustness
- Gain/Phase Crossover Freq – Natural Frequency

Effect of Poles and Zeros in Transfer Function

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7-8-2021

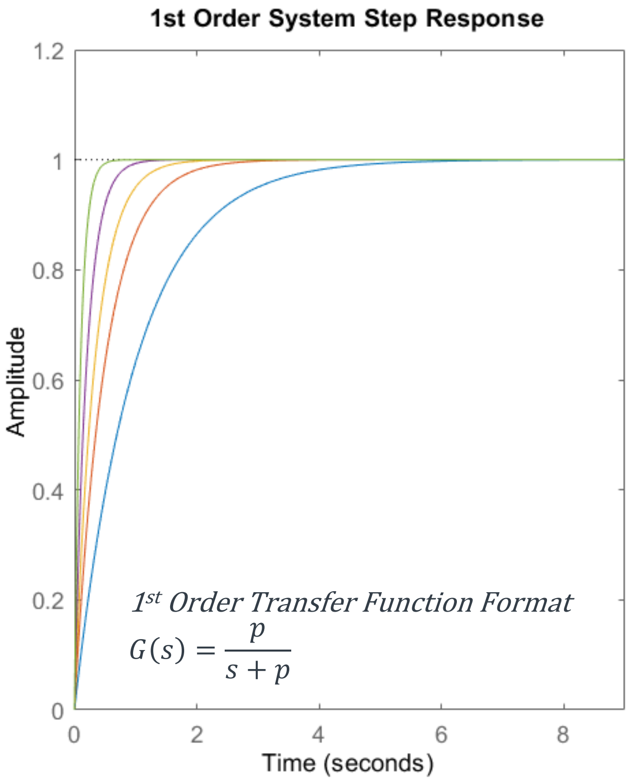
Effect of Poles in 1st and 2nd Order System

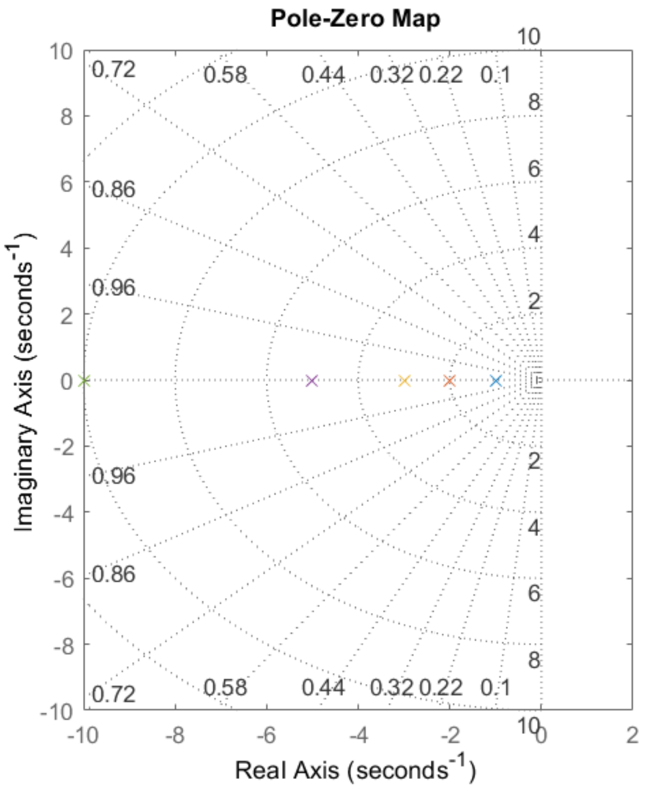
First_Order_System.m
Second_Order_System.m

First-Order System Response

Summary

- Stable First Order System with one LHP (Left Half Plane) pole
- Magnitude of pole determine system response rate

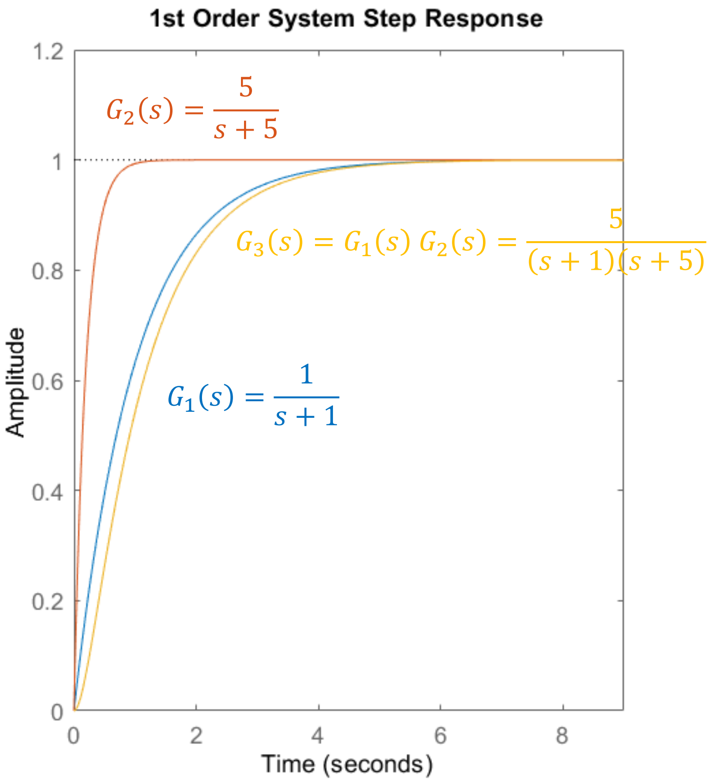


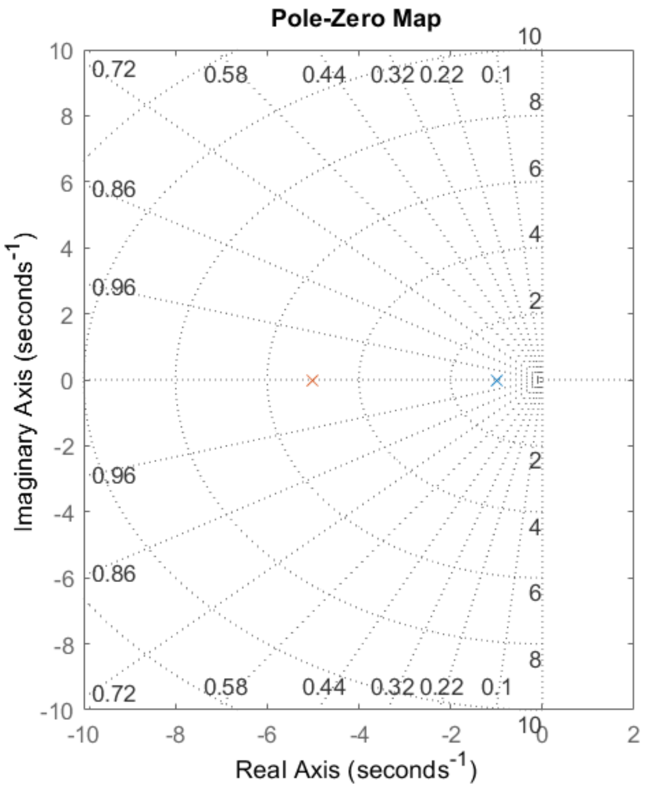


First-Order System Response - Cascade

Summary

- For cascade system, if real part magnitude of two poles are far away, less magnitude is always dominate

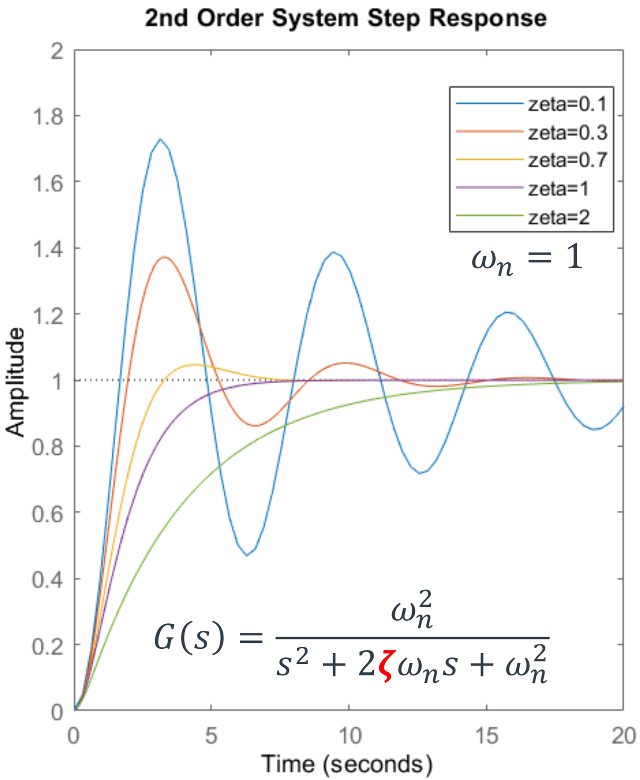


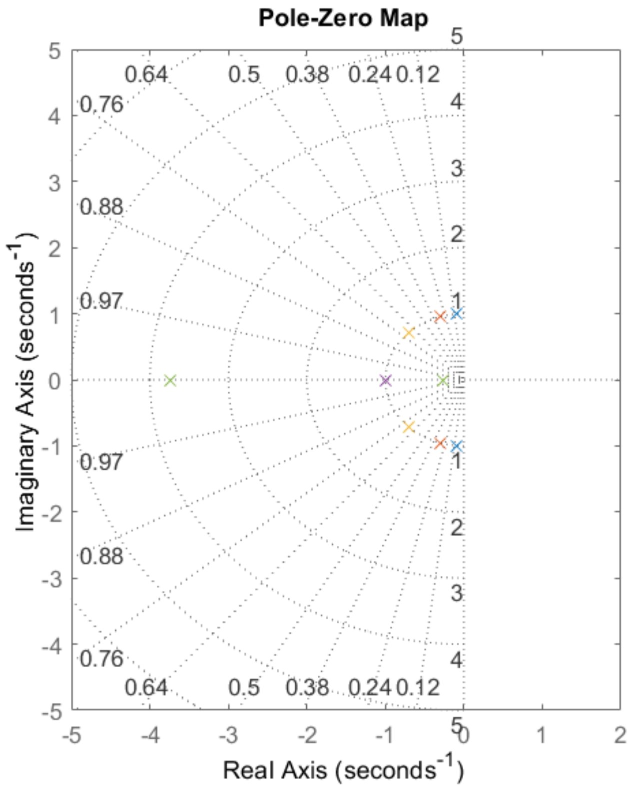


Second-Order System Response – Effect of Damping Factor ζ

Summary

- Damping factor (ζ) affect overshoot and ringing
- ζ within 0.7 to 1 yield best response with minimizing overshoot
- $\zeta > 1$ will generate 2 poles without imaginary part, which is equivalent two 1st order system in cascade

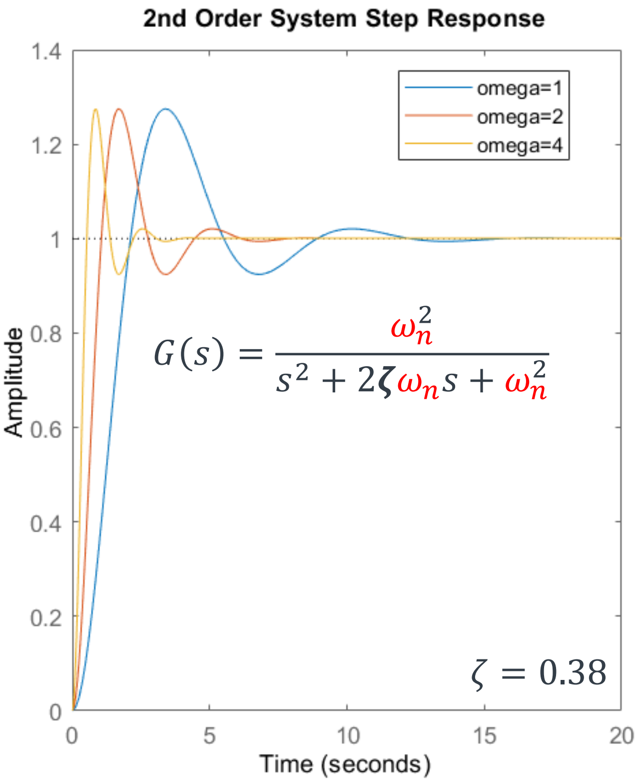


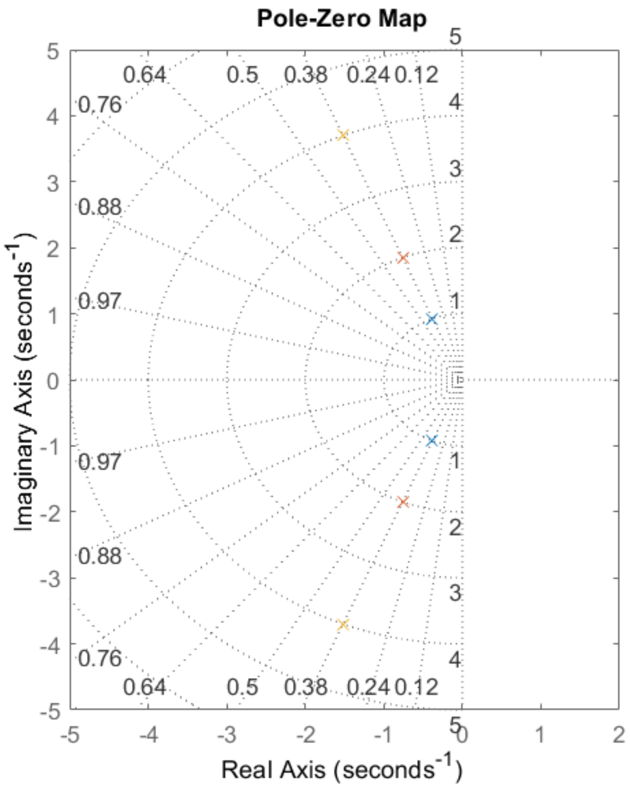


Second-Order System Response – Effect of Natural Freq ω_n

Summary

- Natural Frequency (ω_n) affect system speed
- Natural frequency has no impact on overshoot or undershoot magnitude

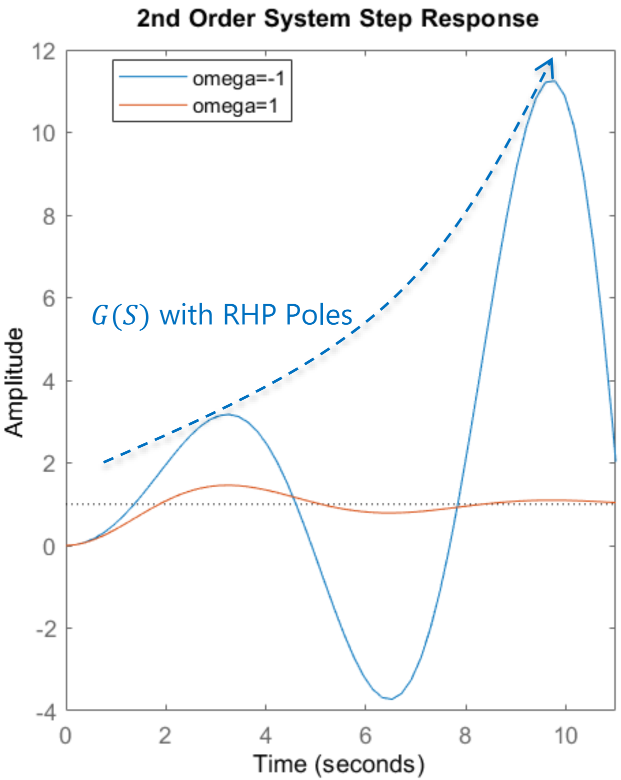


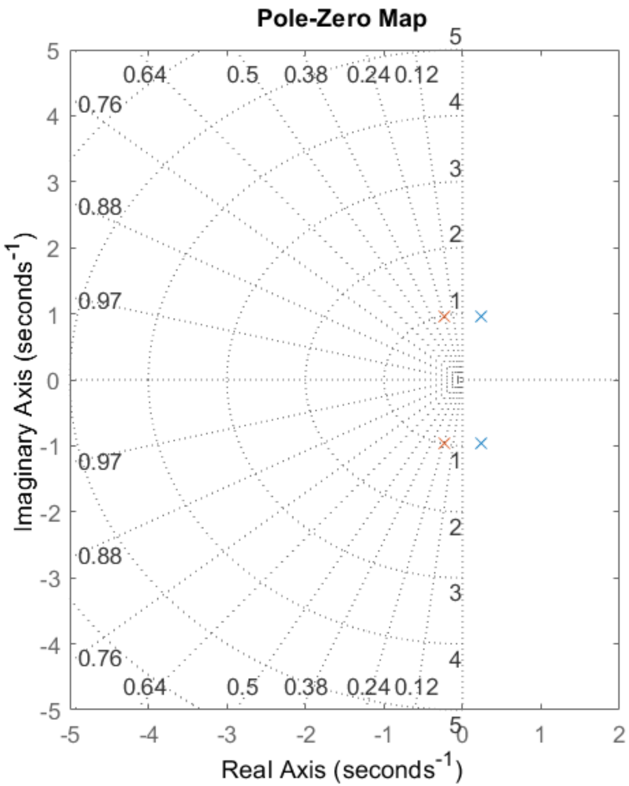


Effect of RHP Poles

Summary

- System response cannot converge with RHP Poles – Unstable System





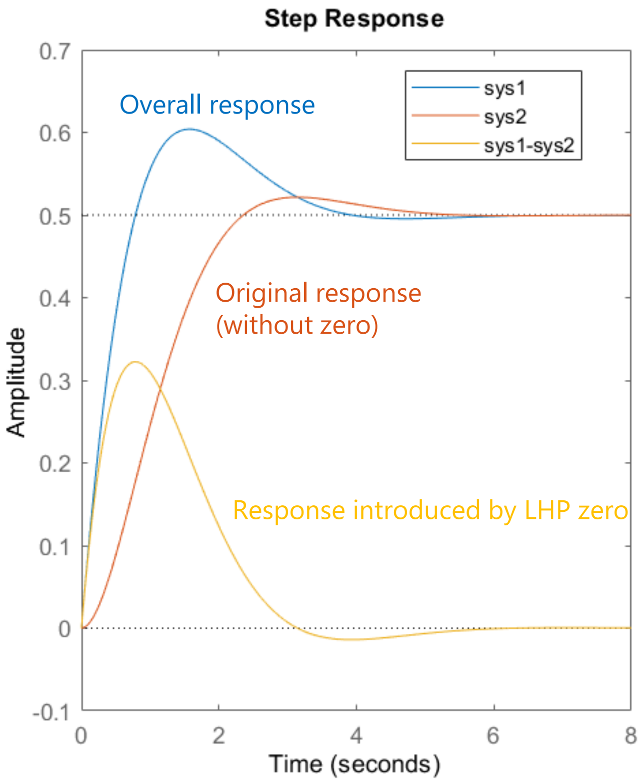
Effect of LHP and RHP Zero

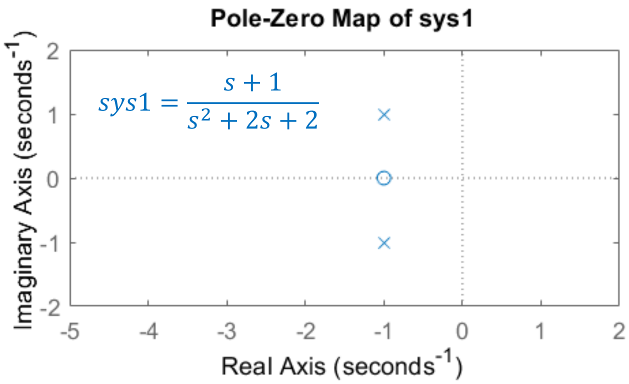
Effect_Of_Zeros.m

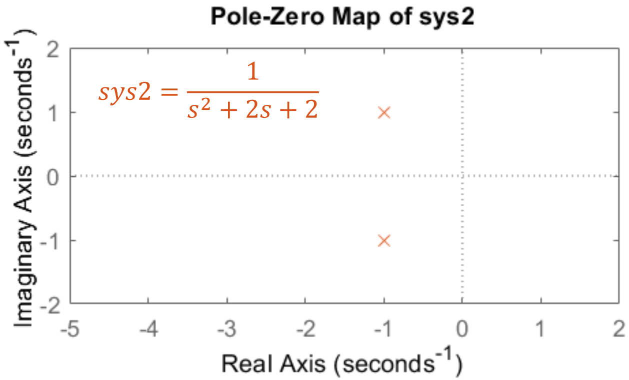
Effect of LHP Zero – Part 1

Summary

- The major effect of the LHP zero is to increase overshoot



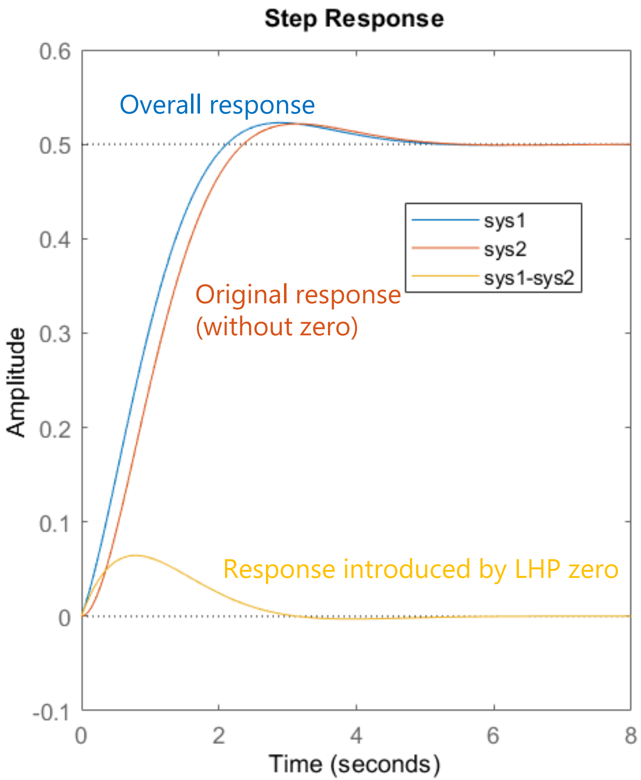


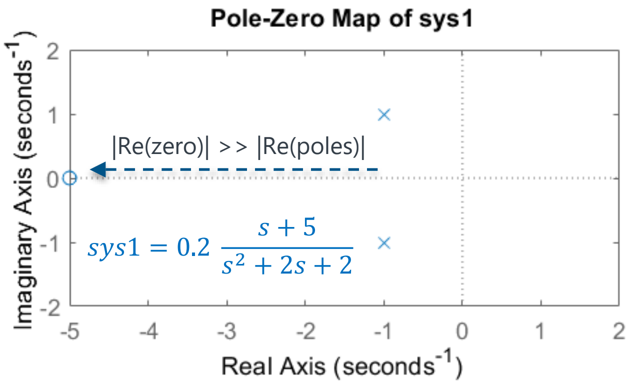


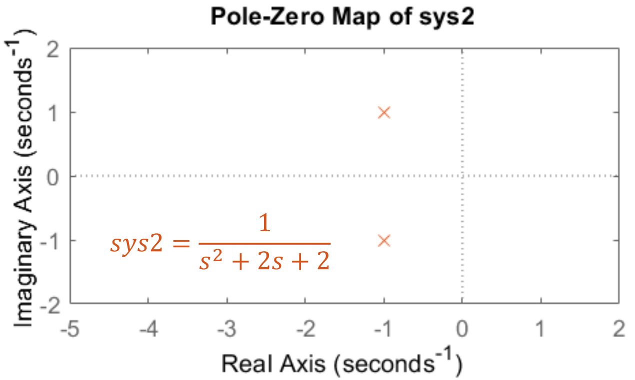
Effect of LHP Zero – Part 2

Summary

- If the zero is far away from real part of complex poles, effect is negligible



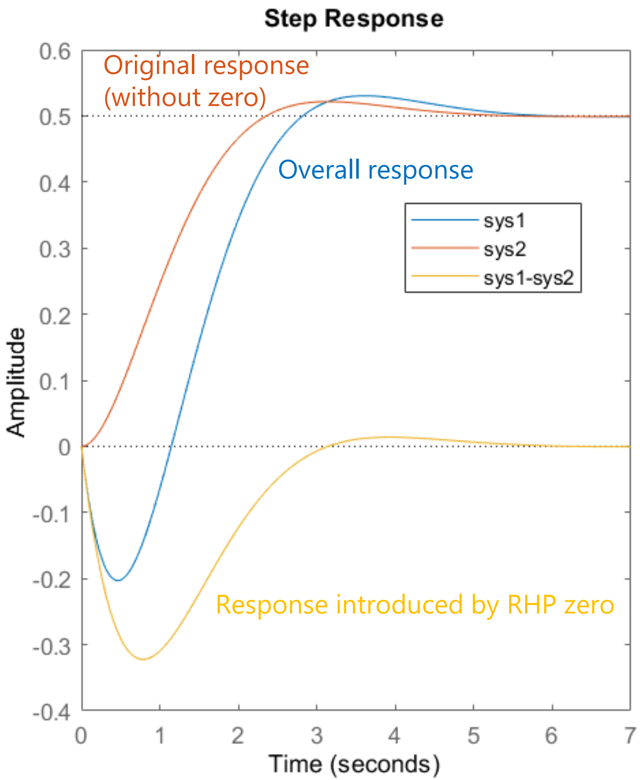


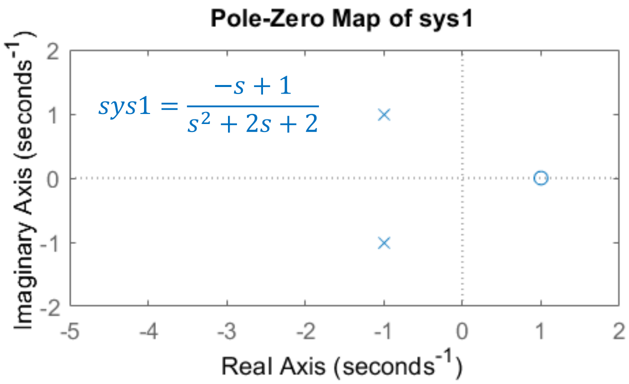


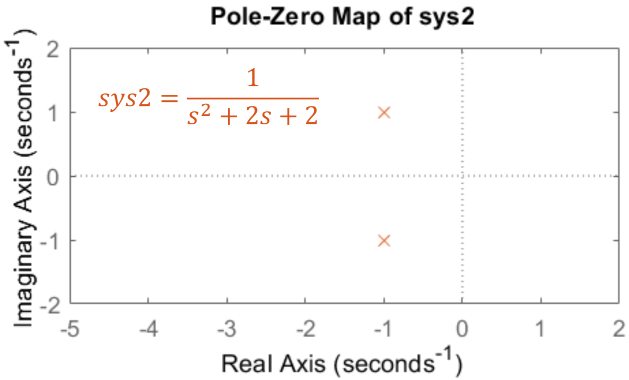
Effect of RHP Zero

Summary

- The major effect of the RHP zero is to cause an initial undershoot (i.e. the step response starts in wrong direction)
- This represents a system when increase its control input, its output will firstly go into a reverse direction



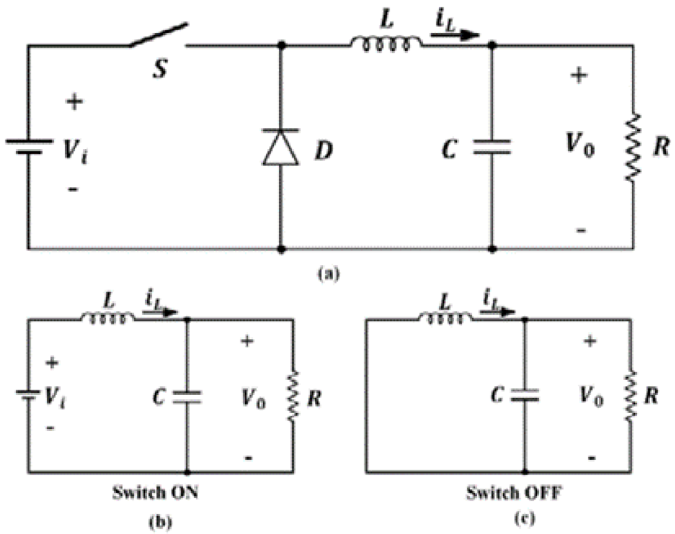


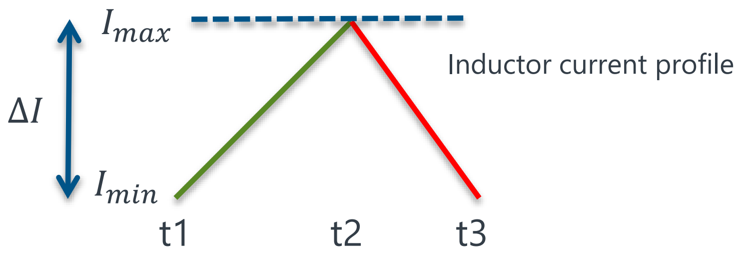


Converter Transfer Function

Kelvin Leung
7-8-2021

Buck Converter





- Switch ON
  - $V_L = L\frac{dI}{dt}$
  - $V_{in} - V_o = L\frac{\Delta I}{t_{on}} = L\frac{\Delta I}{d\,T_s}$
  - $\frac{L\,\Delta I}{T_s} = d(V_{in} - V_o)$

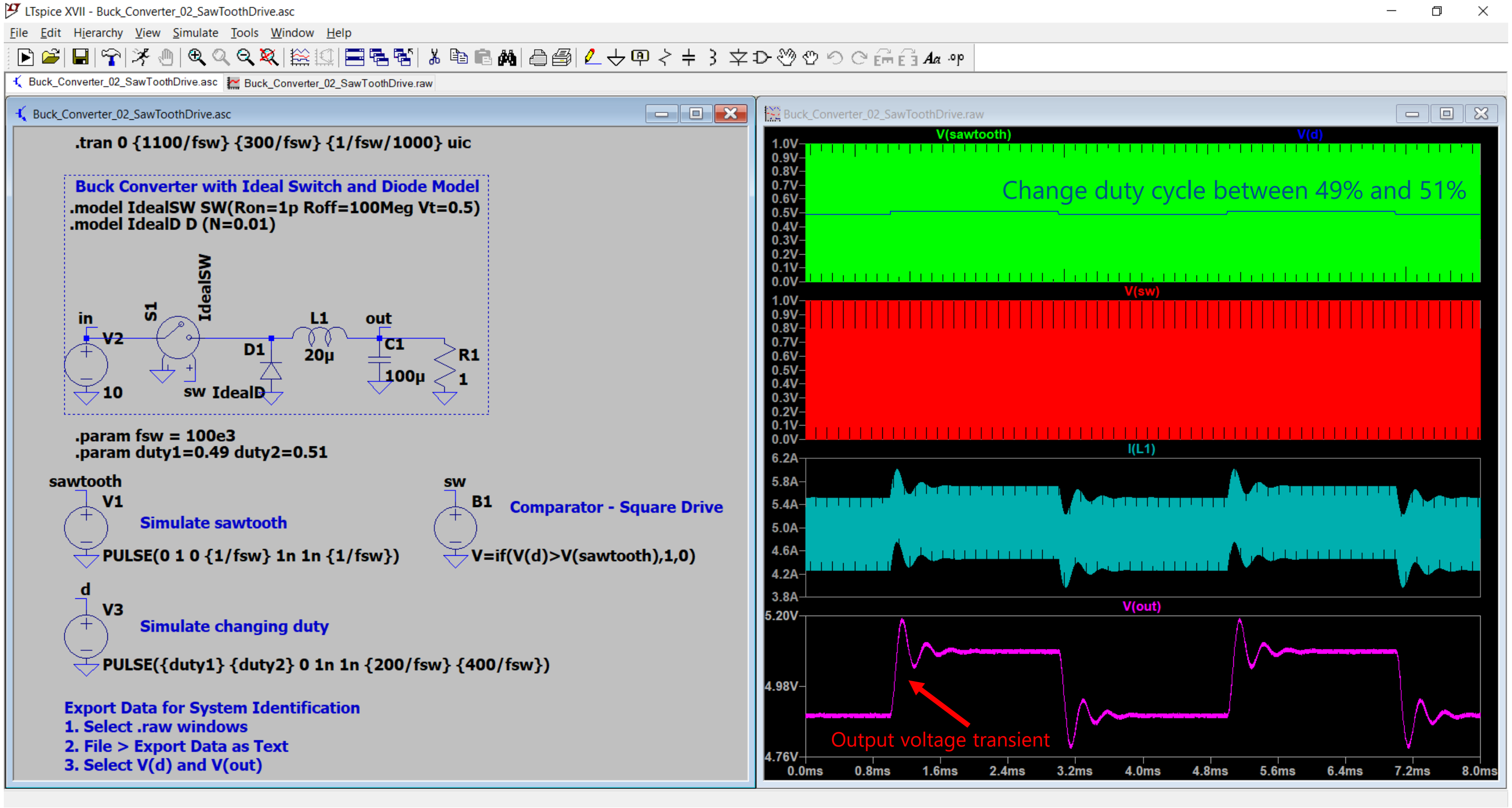
$$\frac{dI}{dt} = \frac{I_{max} - I_{min}}{t2 - t1} = \frac{\Delta I}{t_{on}}$$

- Switch OFF
  - $V_L = L\frac{dI}{dt}$
  - $0 - V_o = L\frac{-\Delta I}{t_{off}} = L\frac{-\Delta I}{(1-d)\,T_s}$
  - $\frac{L\,\Delta I}{T_s} = (1-d)V_o$

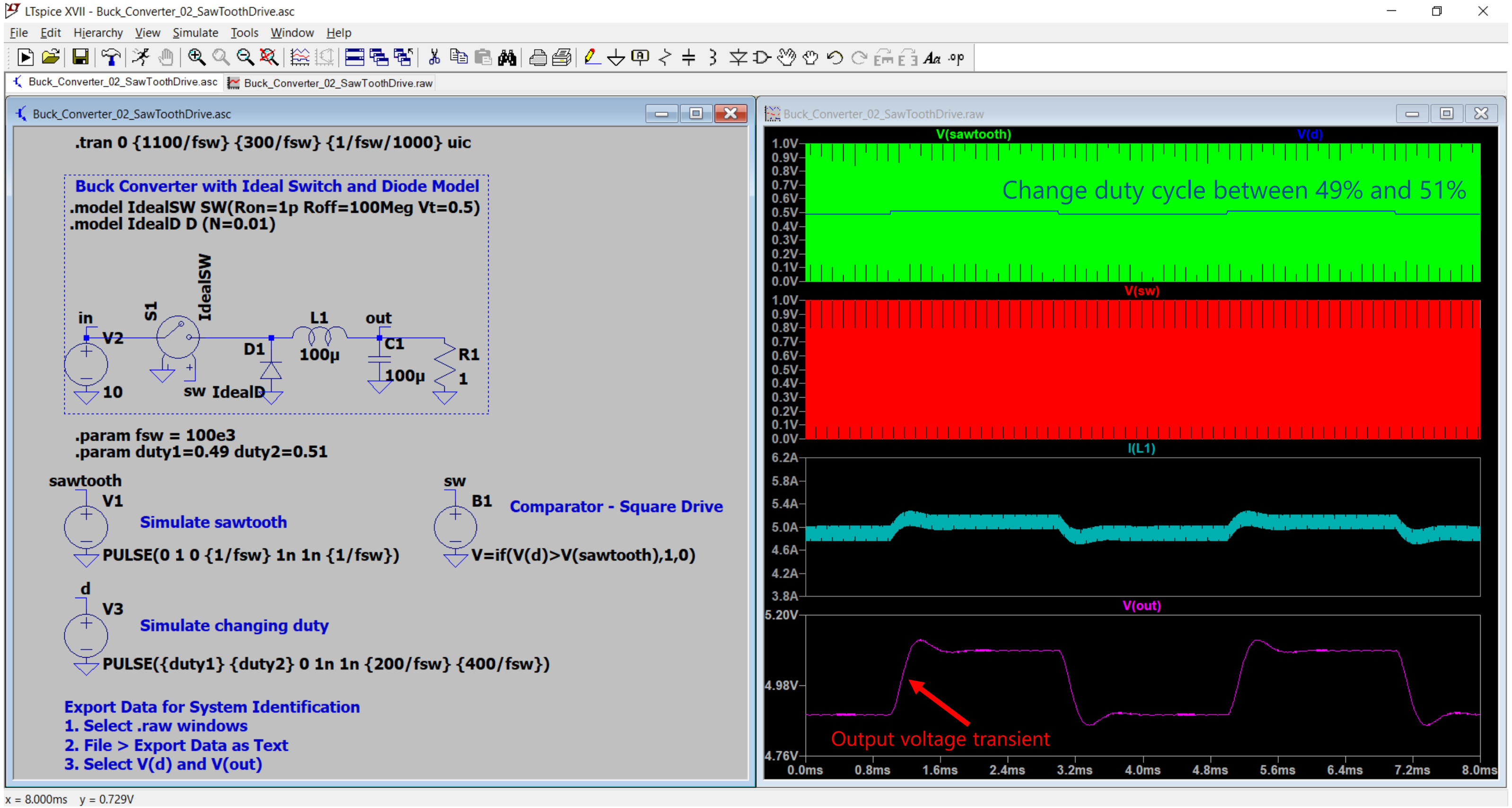
$$\frac{dI}{dt} = \frac{I_{min} - I_{max}}{t3 - t2} = \frac{-\Delta I}{t_{off}}$$

- Combine formula
  - $d(V_{in} - V_o) = (1-d)V_o$
  - $\frac{V_o}{V_{in}} = d$: steady state is not affected by L or C
- Question
  - What is the effect of having different L and C ?

Buck Converter with L = 20uH, C=100uH, R = 1 Ohm



Buck Converter with L = 100uH, C=100uH, R = 1 Ohm

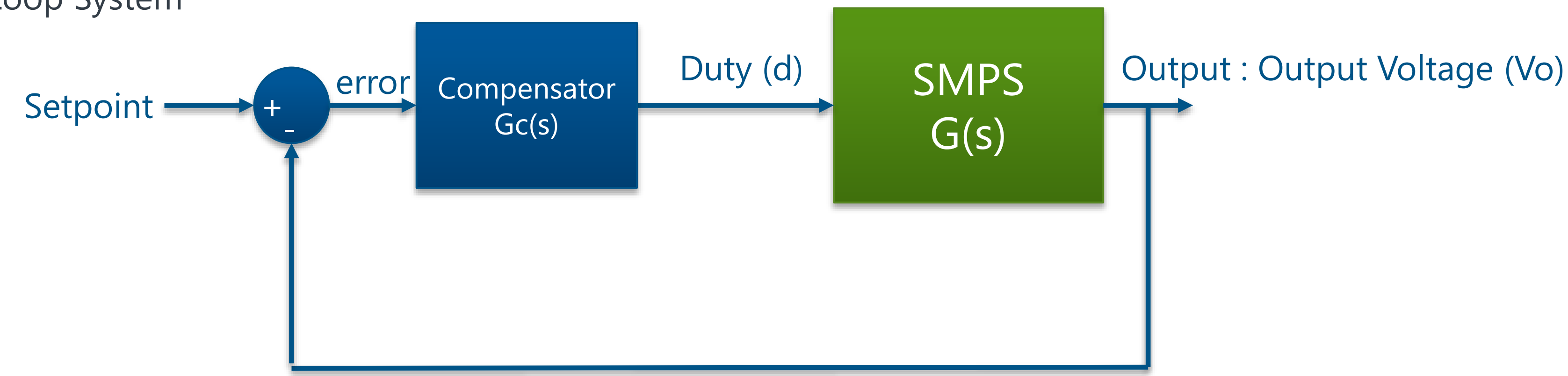


Buck_Converter_02_SawToothDrive.asc

Control System of Switching Mode Power Supply

Open Loop System

Close Loop System



Example : Analog Device LTC3703 Switching Regulator

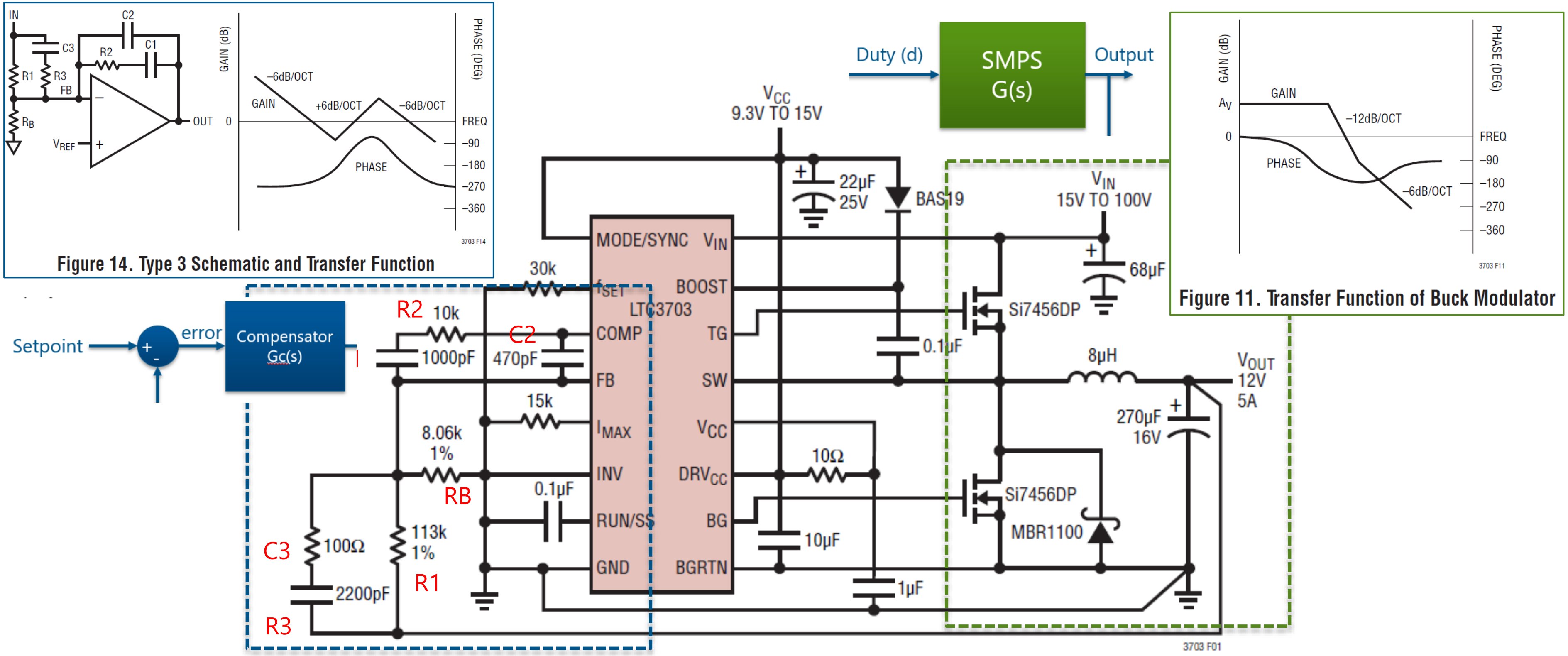


Figure 1. High Efficiency High Voltage Step-Down Converter

Example : Analog Device LTC3703 Switching Regulator

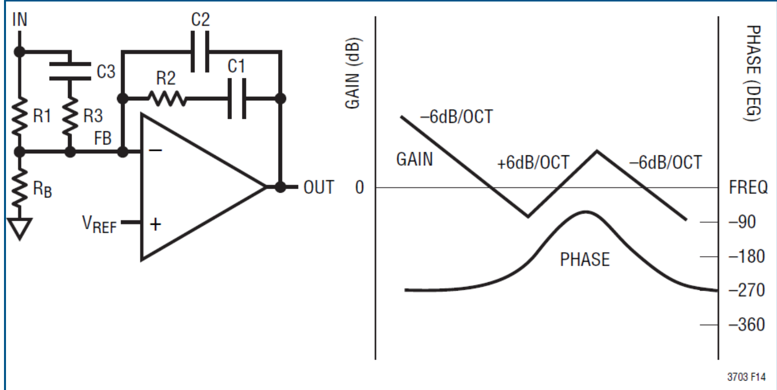
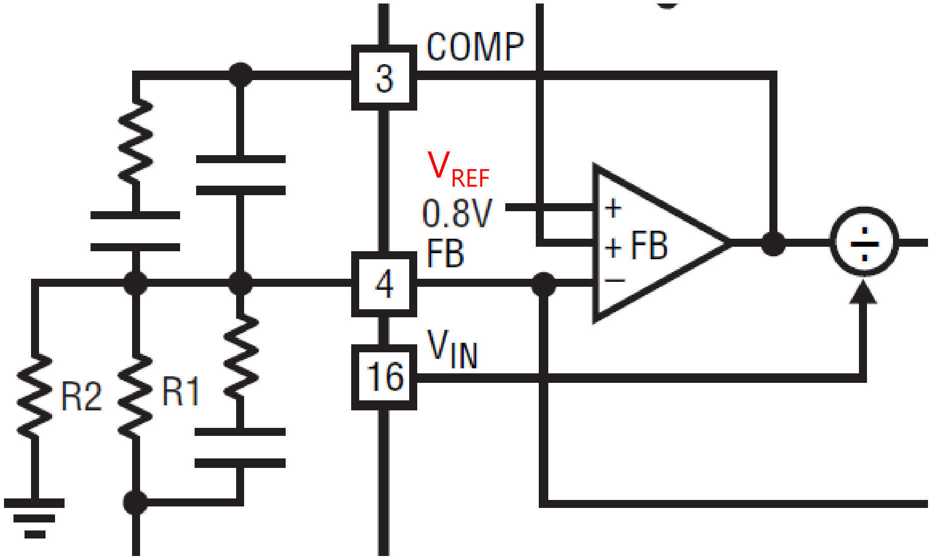


Figure 14. Type 3 Schematic and Transfer Function



First Course on Power Electronics and Drives - Mohan

In Chapter 4

$$L_e = L \text{ (Buck)}; \quad L_e = \frac{L}{(1-D)^2} \quad \text{(Boost and Buck-Boost)} \qquad (4\text{-}14)$$

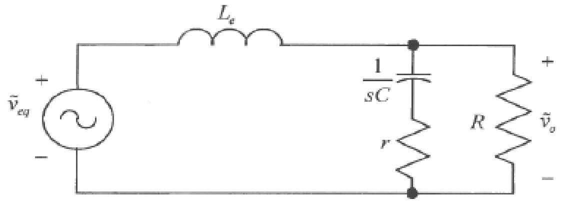


Figure 4-8 Small signal equivalent circuit for Buck, Boost and Buck-Boost converters.

Transfer functions of the three converters in CCM from the Appendix on the accompanying CD are repeated below:

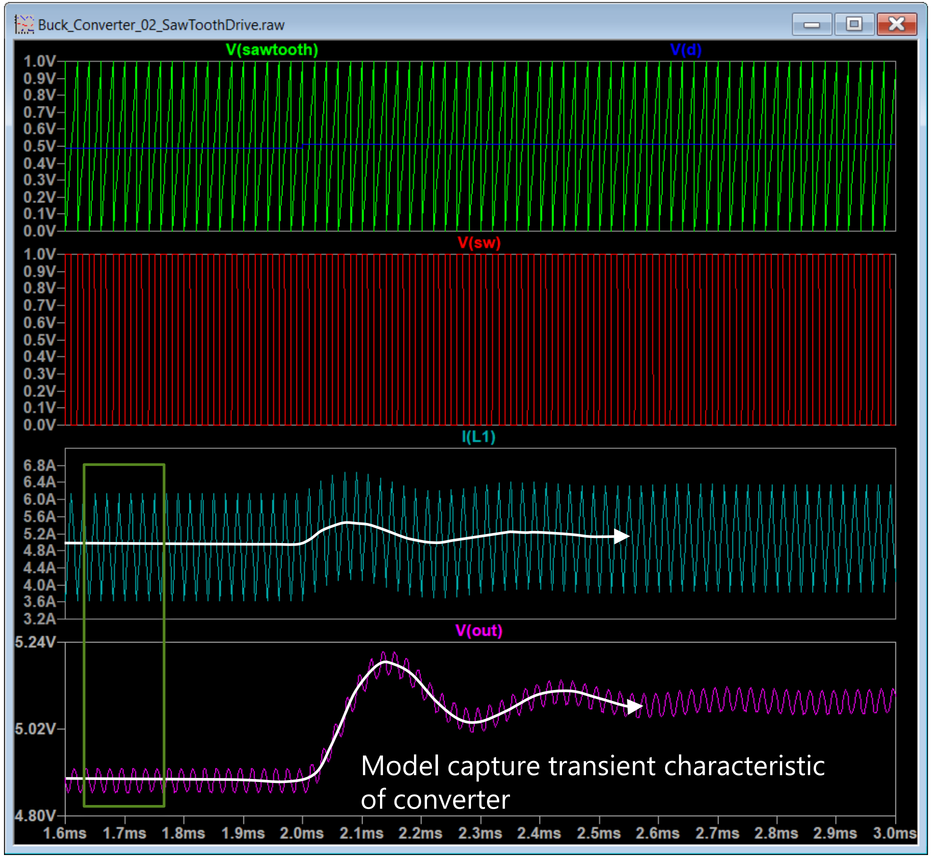
$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{LC} \frac{1+srC}{s^2 + s\left(\frac{1}{RC} + \frac{r}{L}\right) + \frac{1}{LC}} \quad \text{(Buck)} \qquad (4\text{-}15)$$

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{(1-D)^2}\left(1 - s\frac{L_e}{R}\right) \frac{1+srC}{L_e C\left(s^2 + s\left(\frac{1}{RC} + \frac{r}{L_e}\right) + \frac{1}{L_e C}\right)} \quad \text{(Boost)} \qquad (4\text{-}16)$$

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{(1-D)^2}\left(1 - s\frac{DL_e}{R}\right) \frac{1+srC}{L_e C\left(s^2 + s\left(\frac{1}{RC} + \frac{r}{L_e}\right) + \frac{1}{L_e C}\right)} \quad \text{(Buck-Boost)} \qquad (4\text{-}17)$$

Transfer Function in Switching Converter

The modeling ignore behavior within switching action
But to average each switching cycle to calculate its transfer function
e.g. state-space averaging technique



This may need in current mode control

$$G_i = \frac{i_L}{d}$$

$$G_v = \frac{v_o}{d}$$

Buck Converter Transfer Function with L = 20uH, C=100uH, R = 1 Ohm
Calculate from Textbook Formula

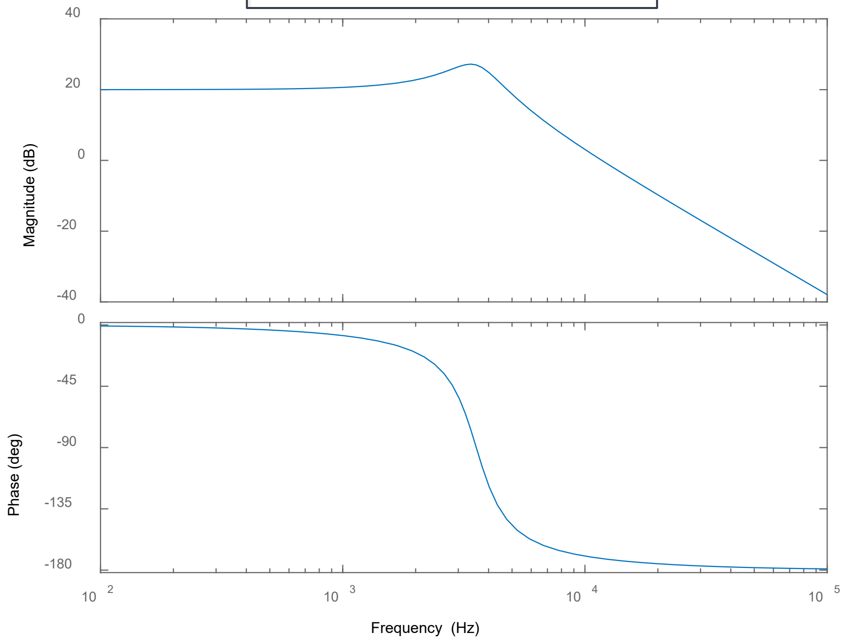
$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_{in}}{LC} \frac{1 + srC}{s^2 + s\left(\frac{1}{RC} + \frac{r}{L}\right) + \frac{1}{LC}} \qquad \text{(Buck)} \qquad (4\text{-}15)$$

```
G =

          5e09
  --------------------
  s^2 + 10000 s + 5e08

Continuous-time transfer function.
```

Matlab Implementation

FirstCourseOnPowerElectroincsAndDrives_CCM_TF_4_8.m

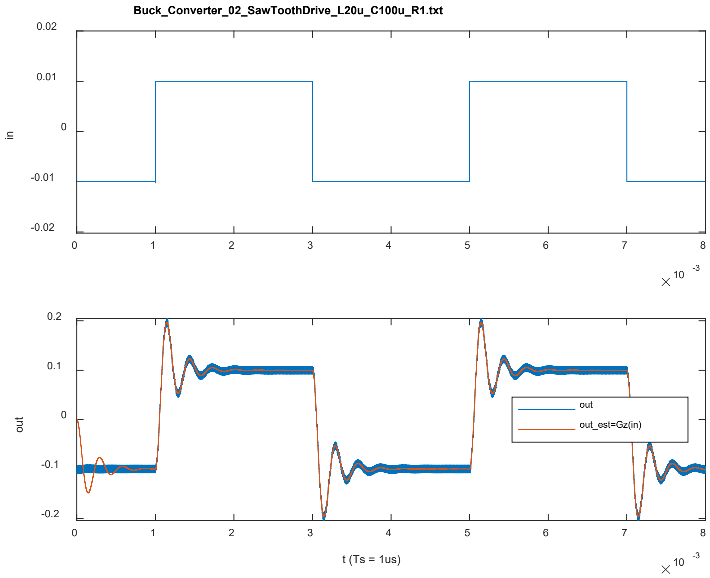
```
L = 20e-6
C = 100e-6
R = 1
Vin = 10
D = 0.5
r = 0

num = Vin/L/C*[r*C 1]
den = [1 (1/R/C+r/L) 1/L/C]

G = tf(num,den)
figure;
h = bodeplot(G)
setoptions(h,'FreqUnits','Hz')
```



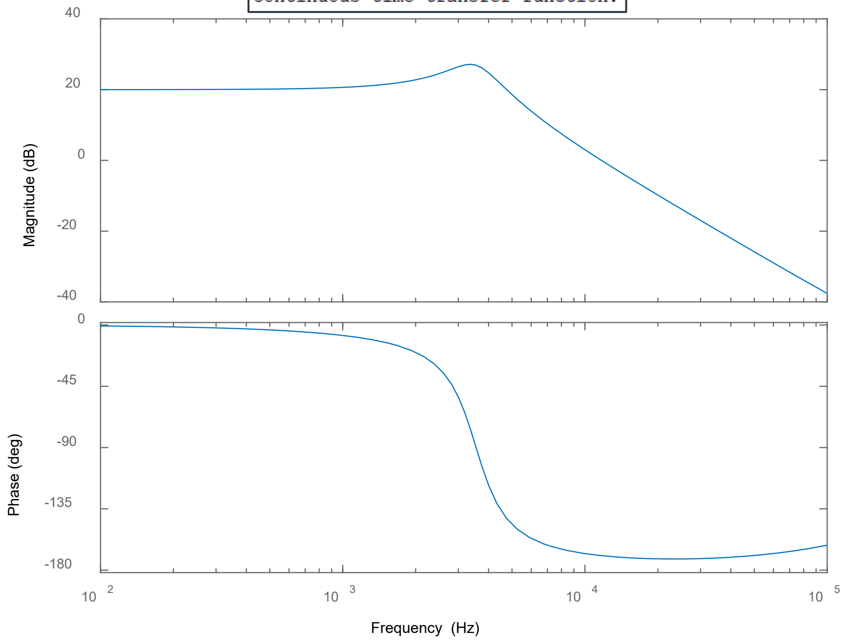
Buck Converter Transfer Function with L = 20uH, C=100uH, R = 1 Ohm Estimated from System Identification Method with LTspice Simluation

ltspice_export_system_identification.m

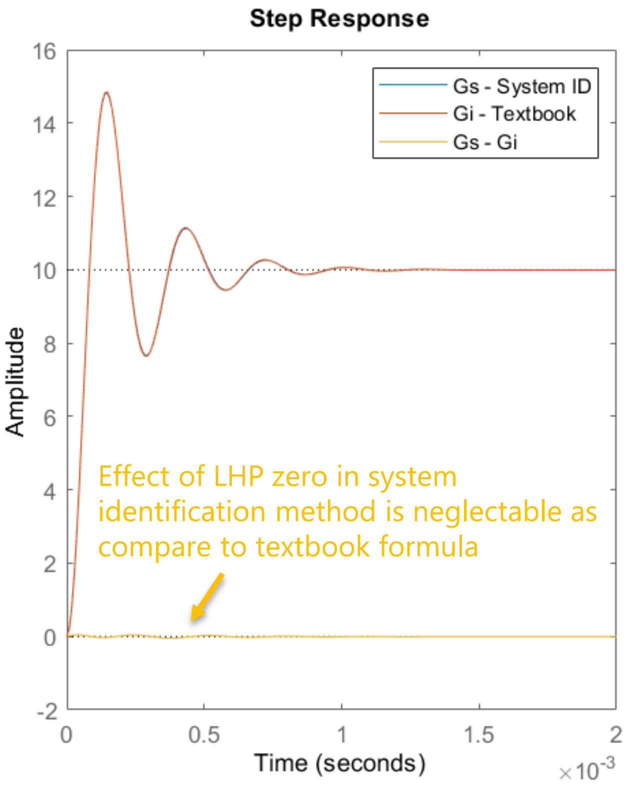
```
% system identification
z = iddata(out,in,Ts);
nb=1;   % order of numerator
nf=2;   % # of poles
nk=1;   % # of delays
m = oe(z,[nb nf nk]);   % output-error polynomial model
Gz = tf(m)              % z-domain transfer function
Gs = d2c(Gz,'zoh')      % s-domain transfer function

out_est = sim(m,in);    % simulate a Simulink model
subplot(2,1,2); hold on;
stairs(t,out_est);
```

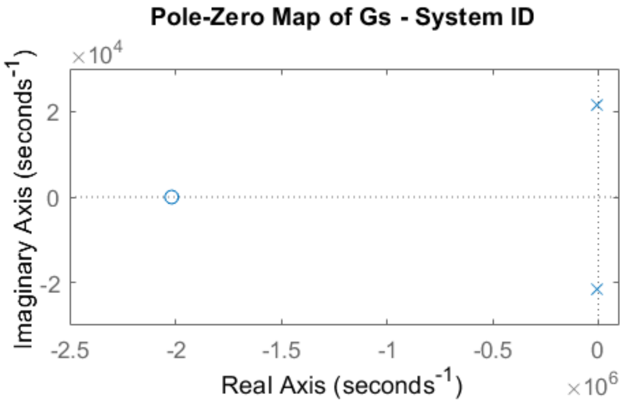


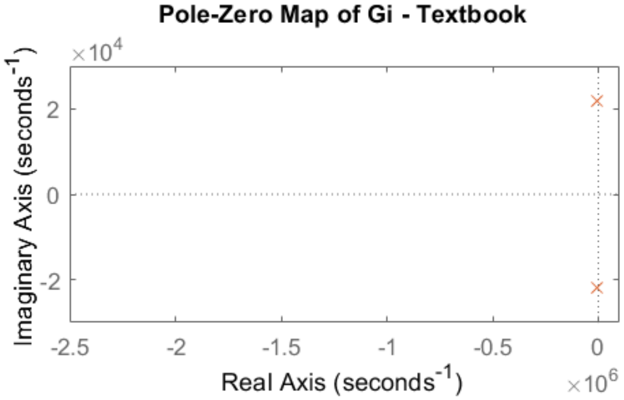

```
Gs =

  From input "u1" to output "y1":
       2475 s + 4.958e09
  ---------------------------
  s^2 + 1.002e04 s + 4.959e08

Continuous-time transfer function.
```



Understanding Poles and Zeros Effect in Transfer Function







Gs is system identification

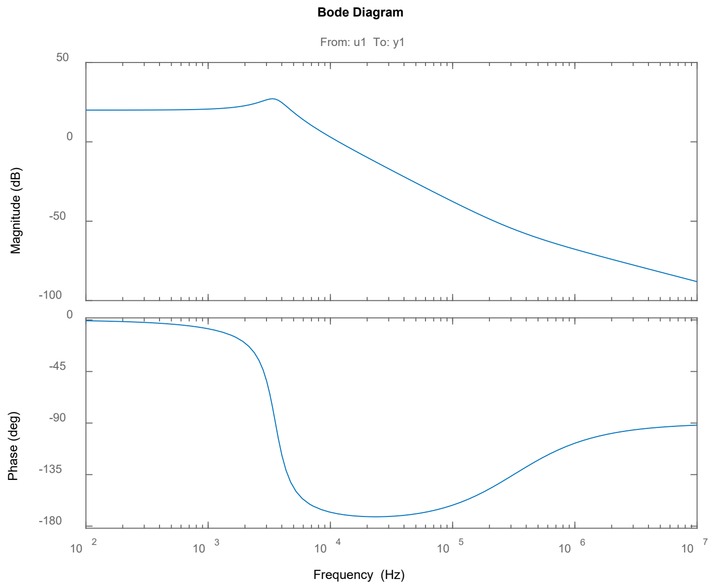
```
Gs =

  From input "u1" to output "y1":
       2475 s + 4.958e09
  ---------------------------
  s^2 + 1.002e04 s + 4.959e08

Continuous-time transfer function.
```

Gi is textbook formula

```
G =

           5e09
  ---------------------
  s^2 + 10000 s + 5e08

Continuous-time transfer function.
```

Compare System Identification Results with AD Datasheet

With System Identification



In Analog Device Datasheet

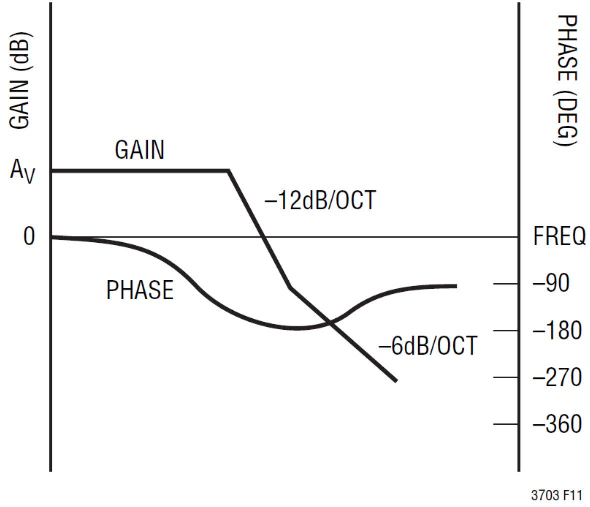
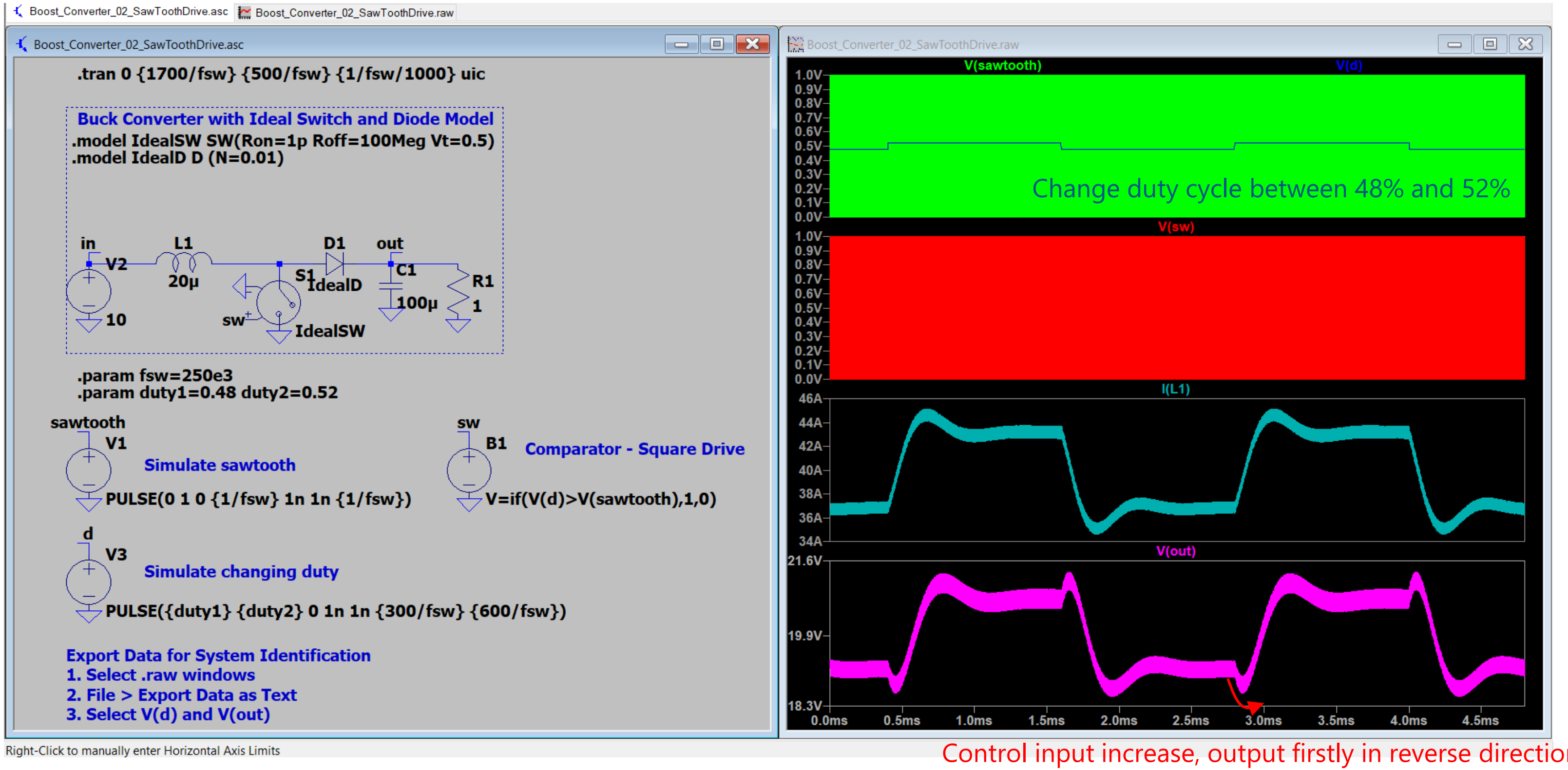


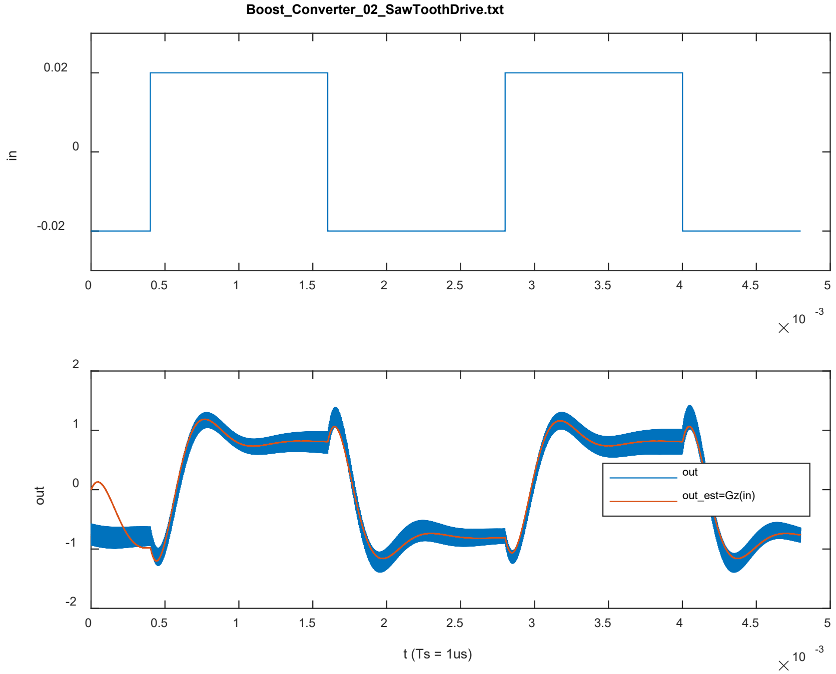
Figure 11. Transfer Function of Buck Modulator

Boost Converter with L = 20uH, C=100uH, R = 1 Ohm

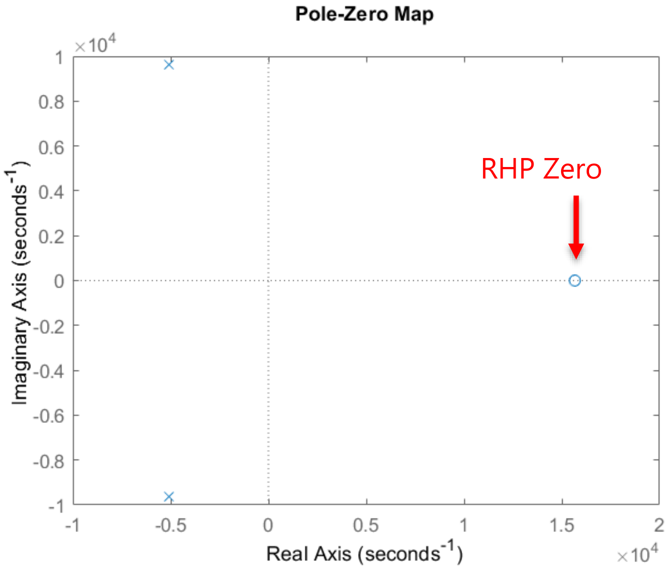


Control input increase, output firstly in reverse direction (i.e. system with RHP Zero)

Boost Converter with L = 20uH, C=100uH, R = 1 Ohm System Idenification Method in Matlab




```
Gs =

  From input "u1" to output "y1":
     -3.045e05 s + 4.77e09
  ---------------------------
  s^2 + 1.017e04 s + 1.184e08

Continuous-time transfer function.
```



Appendix

Fundamentals of Power Electronics - Erickson

In Chapter 8

8.2.2. Transfer functions of some basic CCM converters

Table 8.2. Salient features of the small-signal CCM transfer functions of some basic dc-dc converters

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\frac{V}{D}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
boost	$\frac{1}{D'}$	$\frac{V}{D'}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{L}$
buck-boost	$-\frac{D}{D'}$	$\frac{V}{D\,D'^2}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{D\,L}$

where the transfer functions are written in the standard forms

$$G_{vd}(s) = G_{d0}\frac{\left(1-\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2\right)}$$

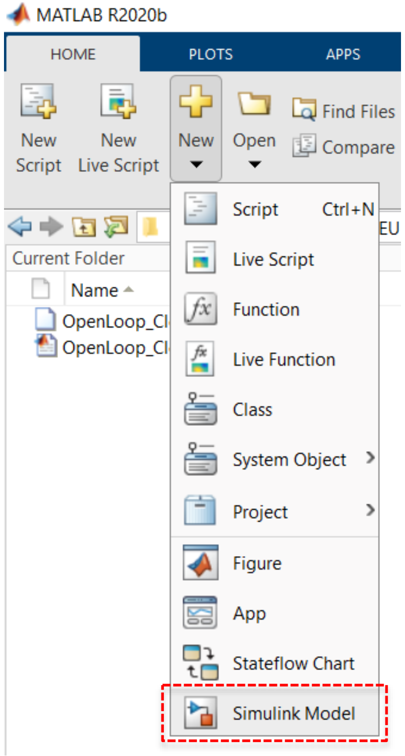
Use of Simulink and Close Loop Feedback with Bode Plot

Kelvin Leung
7-16-2021

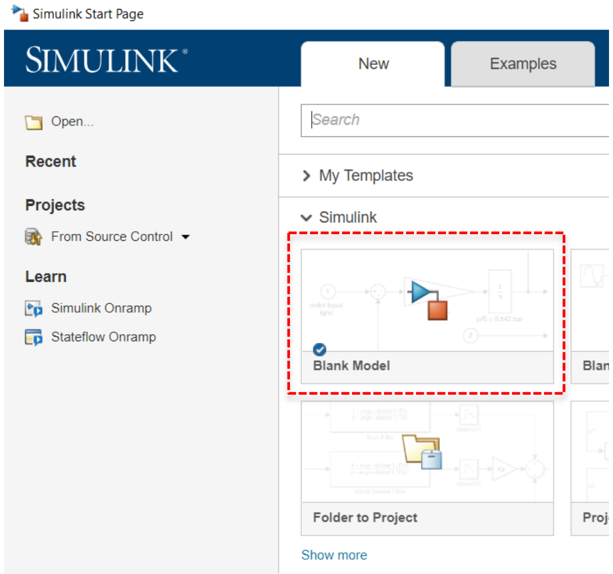
Open Loop Response in Simulink and Matlab Command

Create a Simulink Model

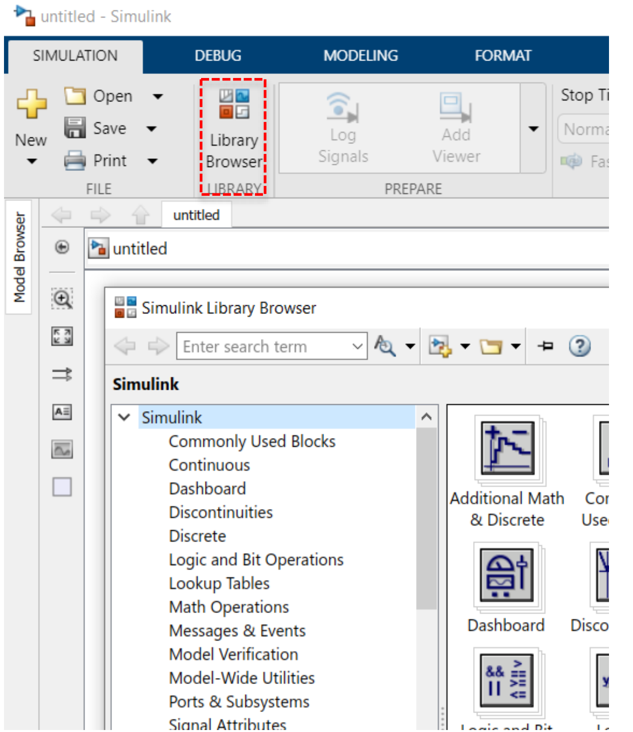
New → Simulink Model



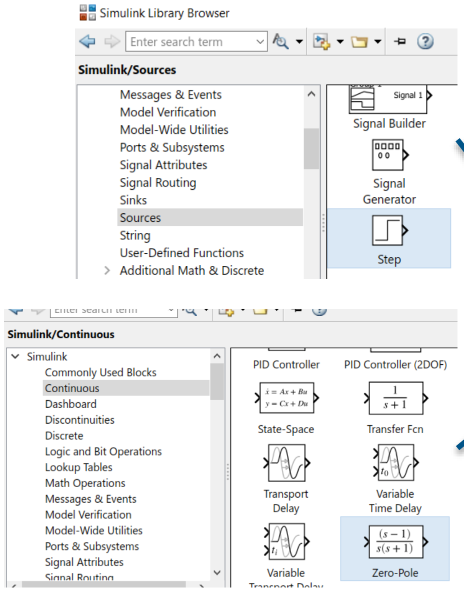
Blank Model

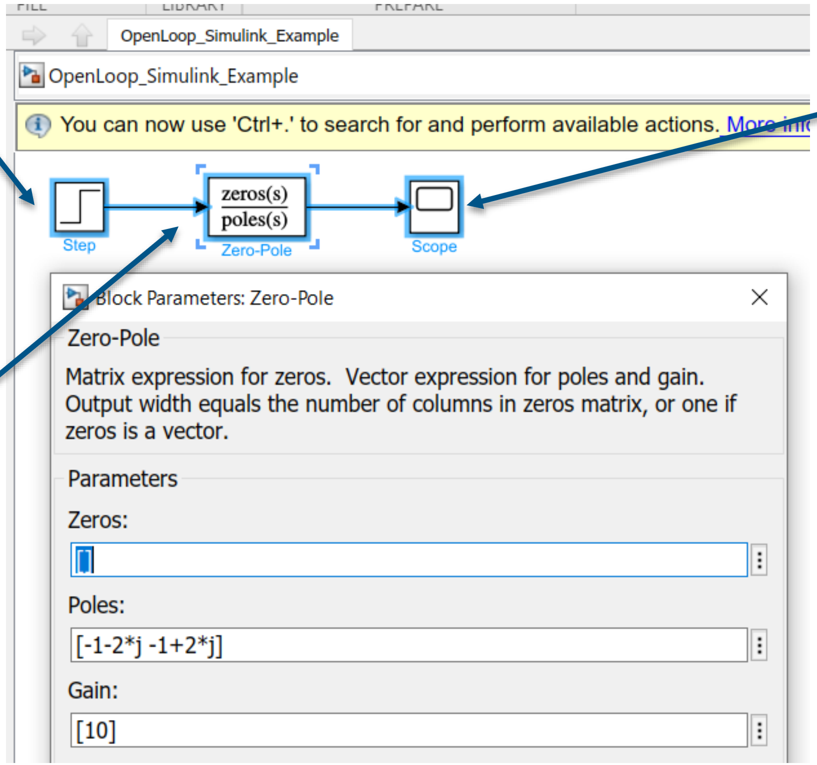


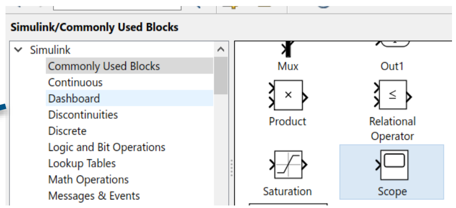
Library Browser

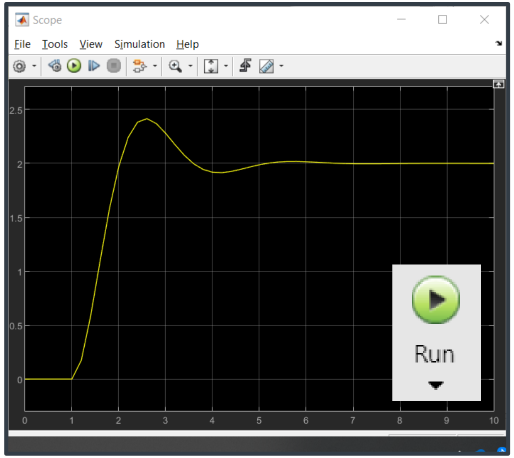


Create a Simulink Open Loop Step Response





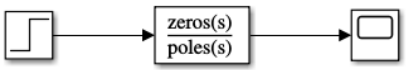




$$G_{open} = \frac{10}{(s+1-2i)(s+1+2i)}$$

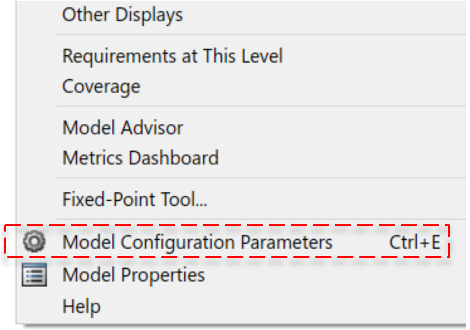
OpenLoop_Simulink.slx

Method to Improve Simulation Result Precision

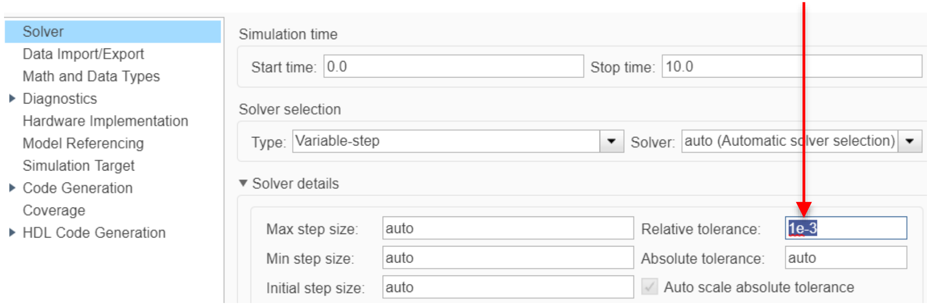


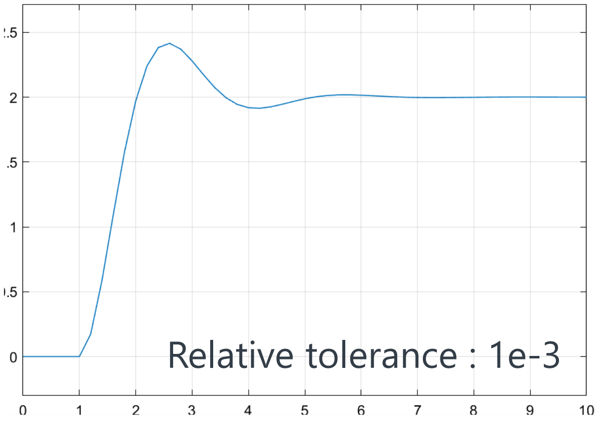
Step 1

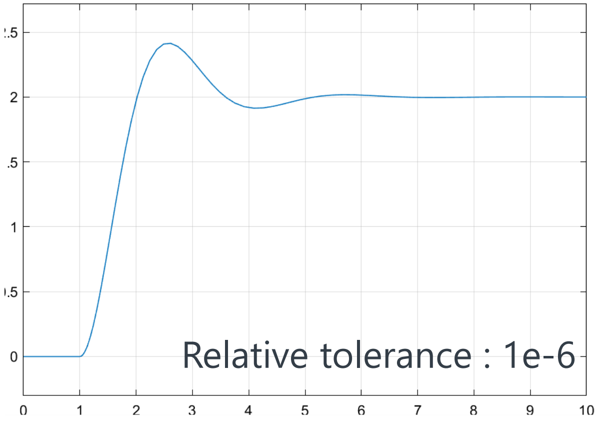
- Right-Click to bring menu
- Select Model Configuration Parameters



Step 2: Reduce Relative tolerance







Equivalent Command for Open Loop Step Response

Control Toolbox Command

Editor - C:\Users\KELVINLEUNG\Desktop\Power Converter Control 2021\04 Basic Open and Close Loop\OpenLoop_Simu

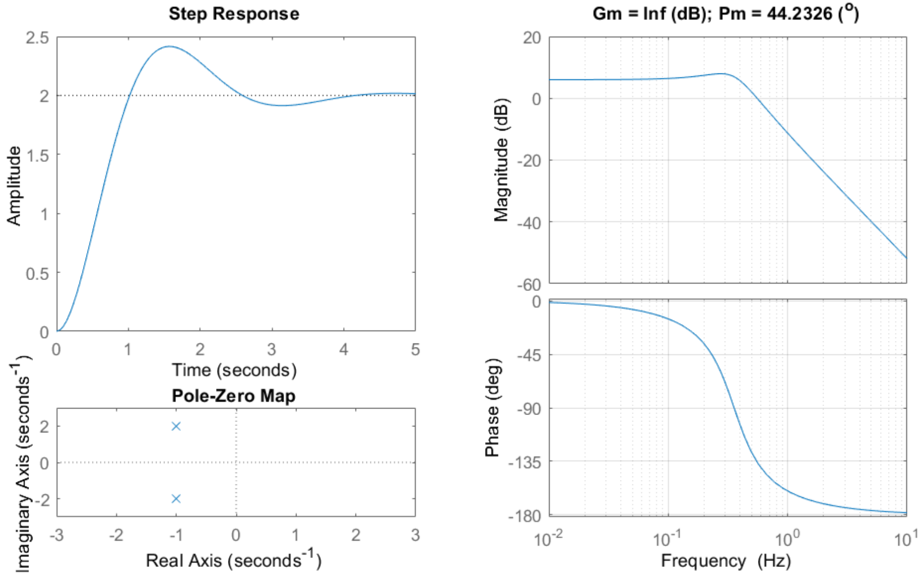
OpenLoop_Simulink_equivalent_Command.m

```
clc;
close all;
clear all;

Gopen = zpk([],[-1-2*1i -1+2*1i],[10]);
display(Gopen)

h = figure;
set(h,'position',[50 50 900 500])
subplot(3,2,[1 3])
step(Gopen);
subplot(3,2,5)
pzmap(Gopen);
axis([-1 1 -1 1]*3);
subplot(3,2,[4 6])
hs = bodeplot(Gopen);
setoptions(hs,'FreqUnits','Hz');
[Gm, Pm] = margin(Gopen);
title(['Gm = ',num2str(20*log10(Gm)),' (dB); Pm = ',num2str(Pm),' (^o)']);
grid on;
```

zpk
Constructs zero-pole-gain model or converts to zero-pole-gain format

step
Step response of dynamic systems

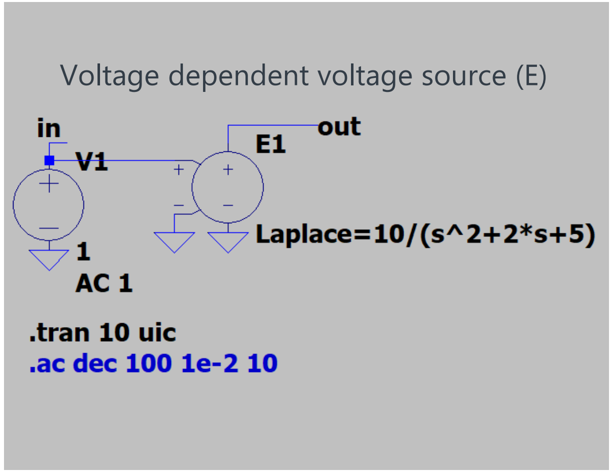
Plot Results



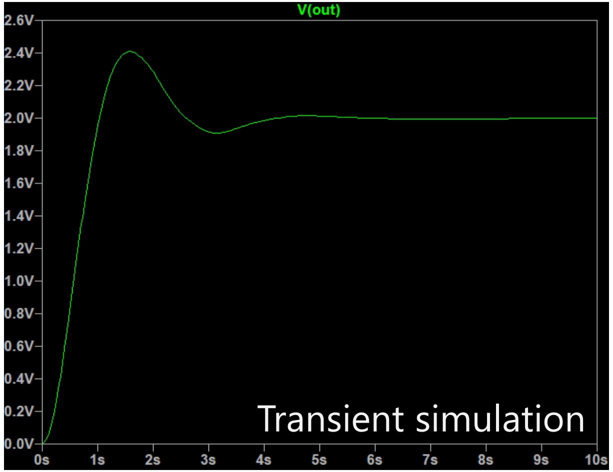
$$G_{open} = \frac{10}{s^2 + 2s + 5} = \frac{10}{(s + 1 - 2i)(s + 1 + 2i)}$$

Equivalent LTSpice for Open Loop Step Simulation

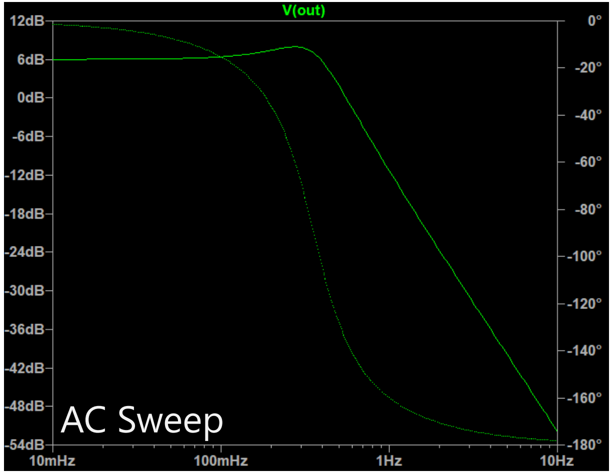
LTspice Schematic



.tran 10 uic

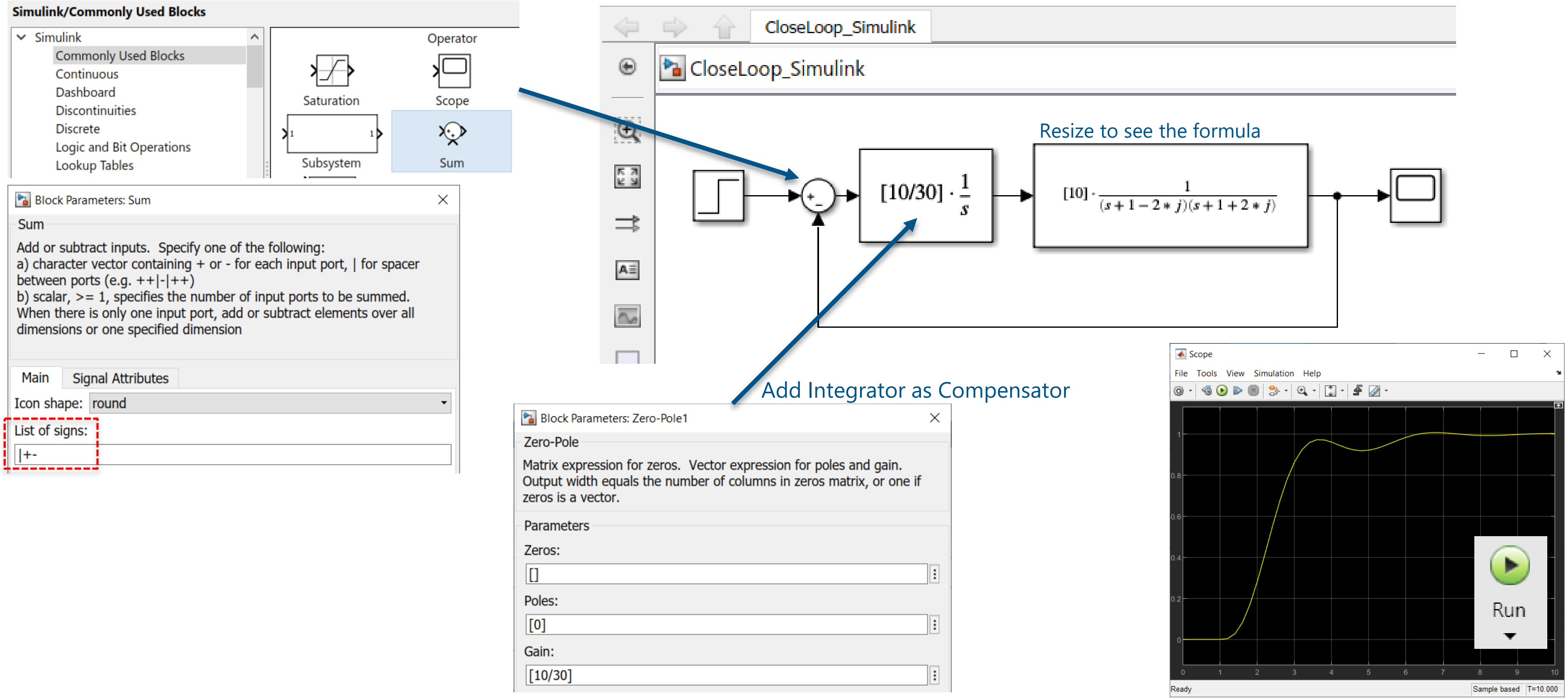


.ac dec 100 1e-2 10



Close Loop Response in Simulink and Matlab Command

Create a Simulink Close Loop Step Response



CloseLoop_Simulink.slx

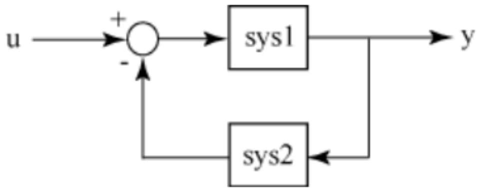
Equivalent Command for Close Loop Step Response

Control Toolbox Command

```
Gplant = zpk([],[-1-2*1i -1+2*1i],[10]);
Gcomp = zpk([],[0],[10/30]);

Gopen = Gplant*Gcomp;
Gclose = feedback(Gopen,1);
display(Gclose)

h = figure;
set(h,'position',[50 50 1200 600])
subplot(3,2,[1 3])
step(Gclose)
```

feedback
Feedback connection to multiple models

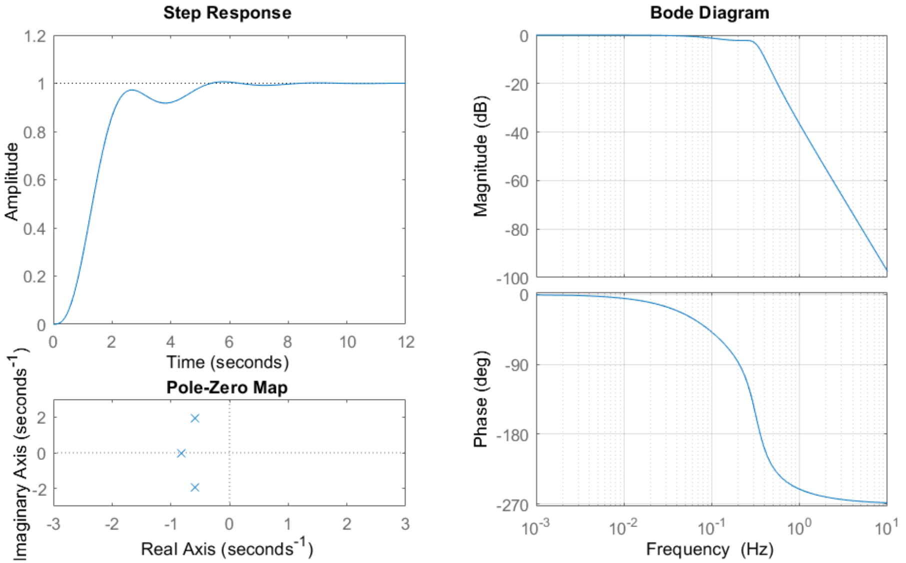
```
sys = feedback(sys1,sys2)
sys1,sys2.
```



$$G_{plant} = \frac{10}{s^2 + 2s + 5}$$

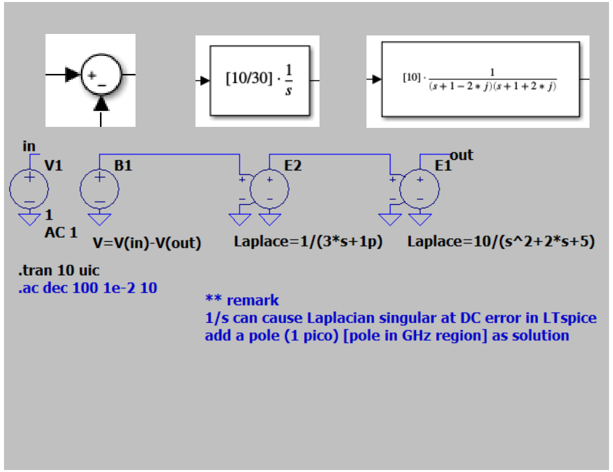
$$G_{comp} = \frac{1}{3s}$$

Plot Results

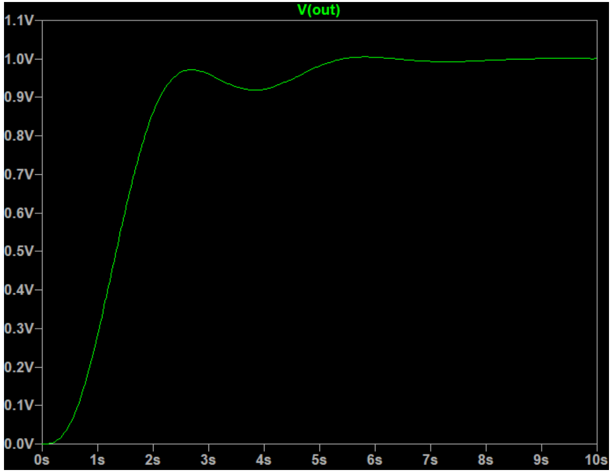


Equivalent LTSpice for Close Loop Simulation

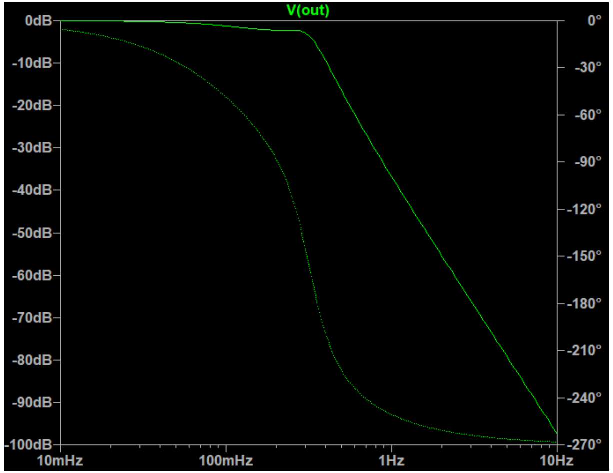
LTspice Schematic



.tran 10 uic



.ac dec 100 1e-2 10

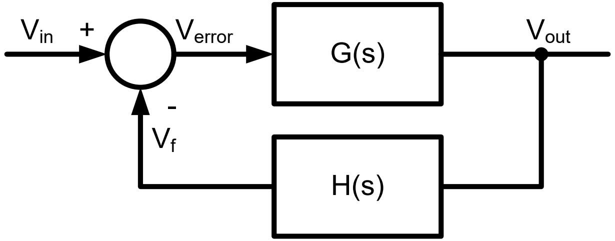


Control Theory in Close Loop Control with Bode Plot Method

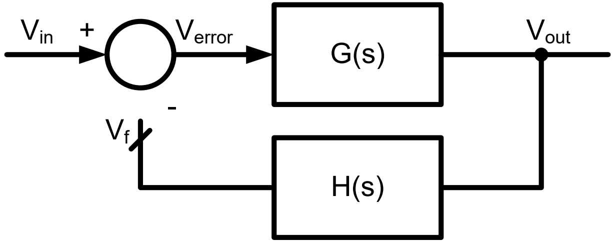
Close Loop Transfer Function

- Close Loop Transfer Function
  - Eqn1 : $V_{out} = G(s)\, V_{error}$
  - Eqn2 : $V_f = H(s)\, V_{out}$
  - By $V_{error} = V_{in} - V_f$
  - $\frac{V_{out}}{G(s)} = V_{in} - H(s)\, V_{out}$
  - $\rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{H(s)+\frac{1}{G(s)}} = \frac{1}{\frac{1+H(s)G(s)}{G(s)}}$
  - $T(s) = \frac{V_{out}}{V_{in}} = \frac{G(s)}{1+G(s)H(s)}$
- Open Loop Transfer Function is defined as
  - $GH(s) = \frac{V_f}{V_{in}} = \frac{V_f}{V_{error}} = G(s)H(s)$
    - where $V_{in} = V_{error}$ if loop is open

Close-Loop Transfer Function



Open-Loop Transfer Function

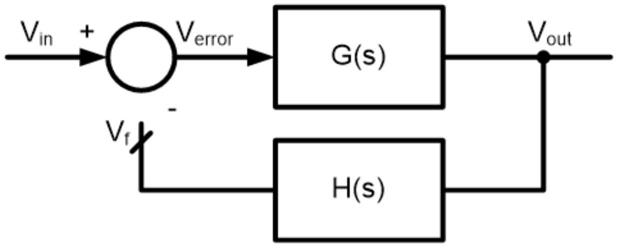


Bode Plot Method

- Bode Plot Method
  - A method to determine close loop performance $T(s) = \frac{G(s)}{1+G(s)H(s)}$ from its open loop transfer function $GH(s) = G(s)H(s)$
- Limitation
  - It can only be used to design close loop feedback from system with stable open loop transfer function, i.e. no RHP poles in $GH(s)$. Otherwise, root locus or Nyquist design technique is required
- Stability Criteria
  - Open Loop Gain of $GH(s)$ at -180° phase angle must <0dB (or 1) to achieve stable close loop $T(s)$

Understand of Physical Meaning of Stability Criteria

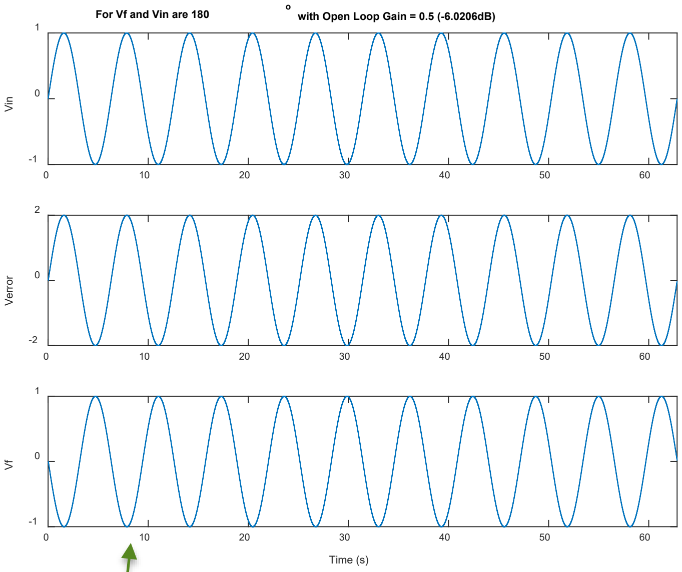
- By imagine the system with discrete timing
  - $V_{in}[n] = \sin(\omega\, t[n])$
  - @ $-180°$ phase
    - $V_f[n] = Gain \times (-V_{error}[n-1])$
  - If closing the loop
    - $V_{error}[n] = V_{in}[n] - V_f[n]$



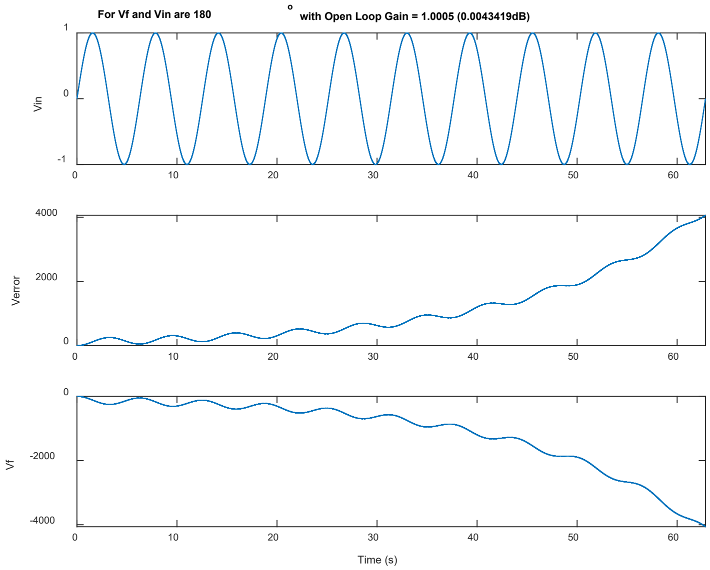

```
t = [0:pi/360:2*pi*10];
omega = 1;

Vin(1) = sin(omega*t(1));
Verror(1) = 0;
Vf(1) = 0;
for n = 2: length(t)
    Vin(n) = sin(omega*t(n));
    Vf(n) = Gain*(-Verror(n-1));
    Verror(n) = Vin(n)-Vf(n);
end
```

Understand of Physical Meaning of Stability Criteria

Gain < 1 (0dB)



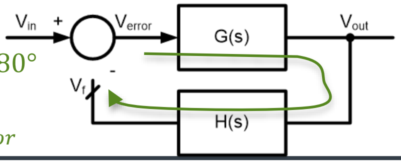
Gain > 1 (0dB)



V_f achieved equilibrium and stable

Open Loop @ 180°
Gain = 0.5

$V_f = -0.5\, V_{error}$

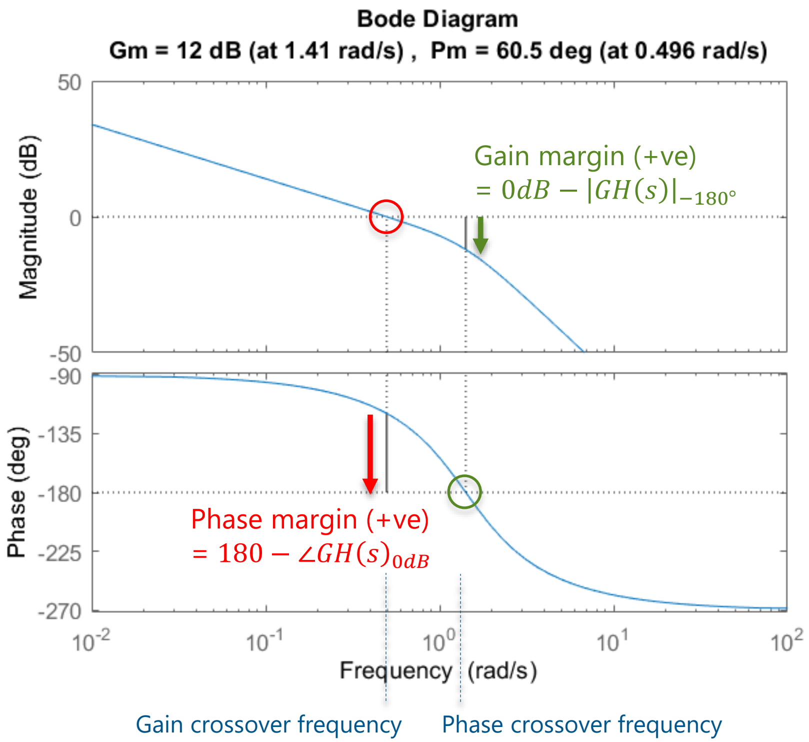


error and feedback keeps amplifing with a positive feedback!!

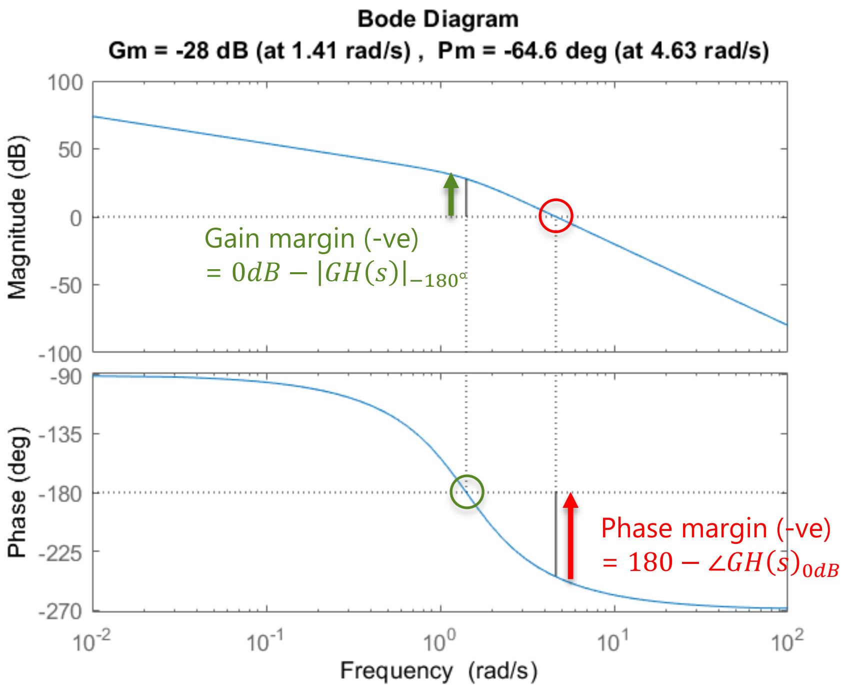
That why open loop gain @ -180 degree must less than 1 (0dB) for a stable system design

Meaning of Gain Margin and Phase Margin

Stable System



Unstable System



gain_phase_margin_basic.m

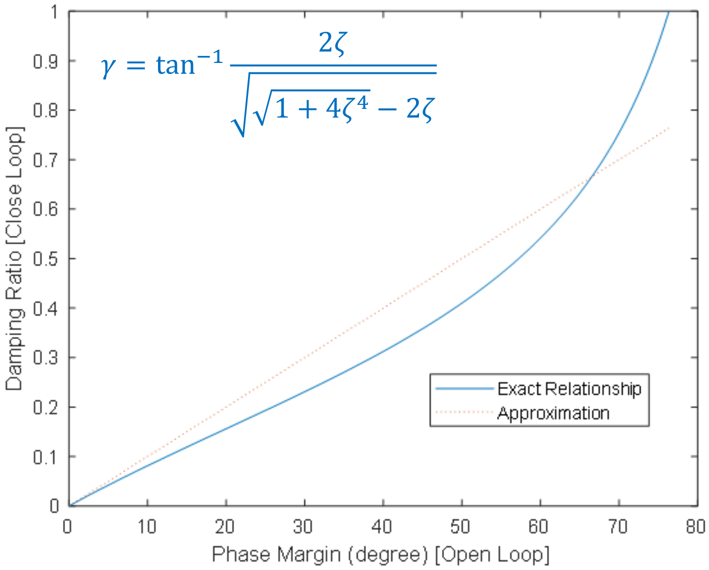
Relationship between Open Loop and Close Loop Performance

- Natural frequency (ω_n)
  - ω_n in closed-loop system is somewhere between the gain crossover frequency and phase crossover frequency in open-loop system.
    - page. 473-474 of "Modern Control Engineering", Ogata, 5th Edition
  - A very rough estimate is that the bandwidth (freq @ -3dB) is approximately equal to the natural frequency.
    - [http://www.engin.umich.edu/class/ctms/freq/freq.htm]

- Damping ratio (ζ)
  - Phase margin in open-loop system has linear relationship with ζ of closed-loop system
  - Exact Formula : Phase Margin (γ) and Damping Ratio (ζ)
    - $\gamma = \tan^{-1} \frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4}-2\zeta}}$
  - Approximation: for $\zeta < 0.6 \rightarrow \zeta = 0.01\,\gamma$

Design Guideline with Bode Plot

- DC Gain : Determine the steady state error
  - Increase of DC Gain, Decrease of Steady State Error (if $Gain \rightarrow \infty$, $V_{error} \rightarrow 0$ if)
- Phase margin : Determine the damping ratio and overshoot
  - Phase margin is normally selected to between 30º-60º.
- Gain margin : Determine the robustness of system
  - To guarantee stability even if the open-loop gain and time constants of the components vary to a certain extend. Normally > 6dB.
- Gain/Phase crossover frequency : Determine the transient response speed
  - Increase the crossover frequency, Increase transient speed.

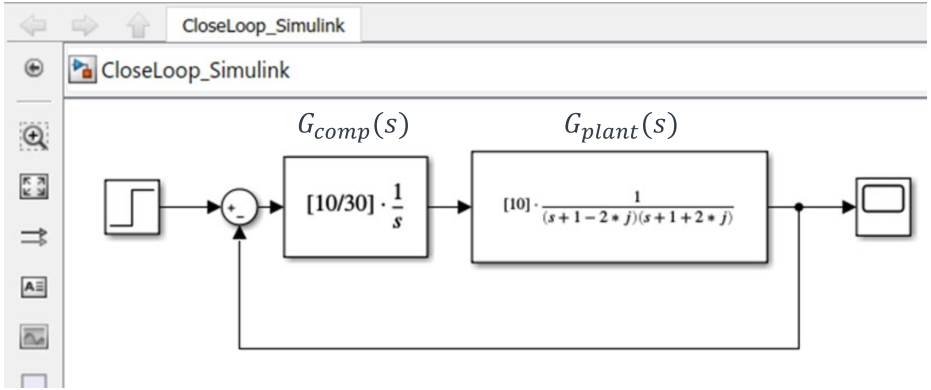


PhaseMargin_DampingRatio_ExactRelationship.m

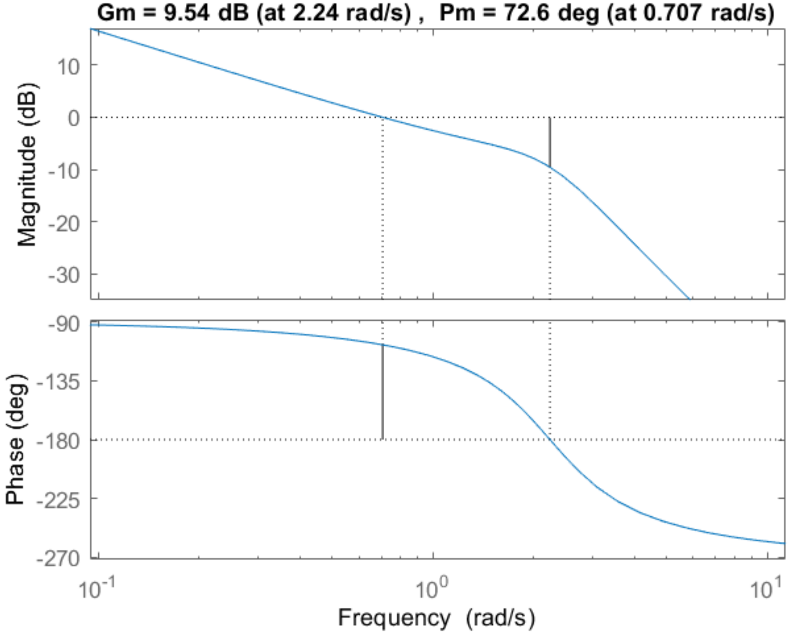
Design Example – Estimate Close Loop from Open Loop TF

- An example to estimate close loop performance from open loop transfer function

- Open Loop Transfer Function
  - $G_{plant}(s) = \frac{10}{(s+1-2j)(s+1+2j)} = \frac{10}{s^2+2s+5}$
  - $G_{comp}(s) = \frac{1}{3s}$
  - $H(s) = 1$
  - $GH(s) = G_{plant}(s)\, G_{comp}(s) H(s)$



Bode Plot of Open Loop System GH(s)

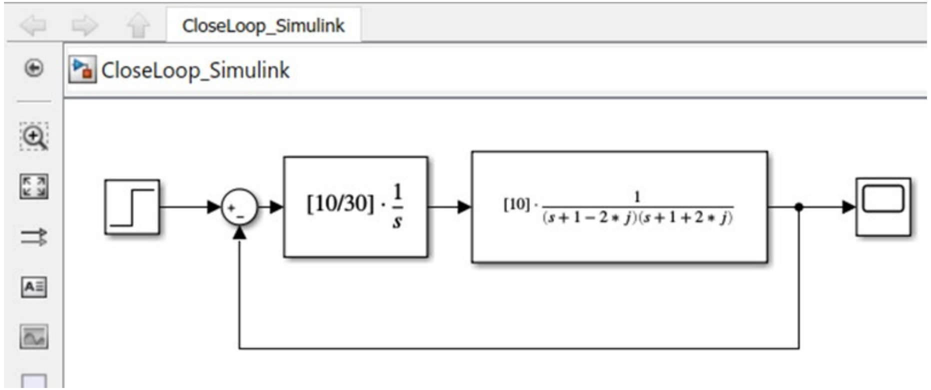


Estimation of Close Loop Performance from GH(s) Bode Plot

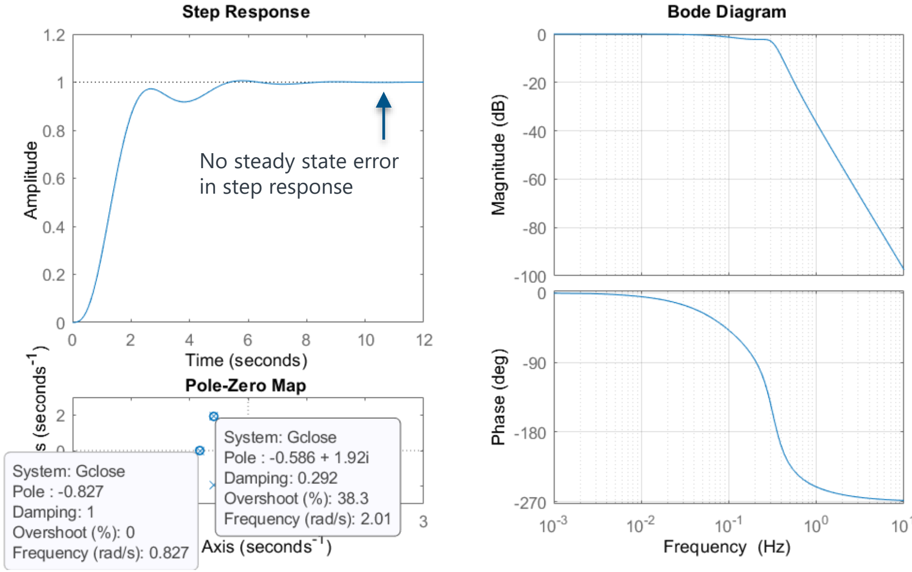
- DC Gain → ∞ : Steady State Error = 0
- PM is 72.6°, damping ratio is ~0.8 (from exact curve)
- From Gain/Phase crossover frequency, natural freq ~ 1 rad/s

Design Example – Close Loop Performance

- Close Loop Transfer Function
  - $T(s) = \frac{G(s)}{1+G(s)H(s)}$

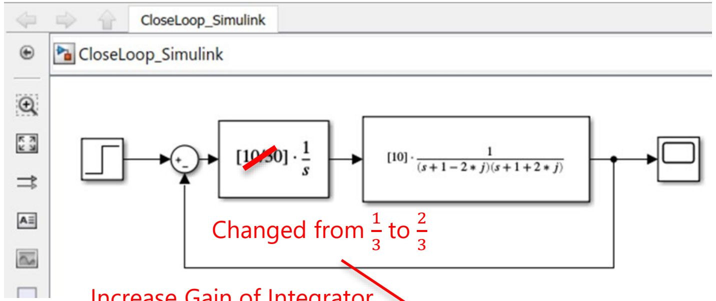


Close Loop System Response



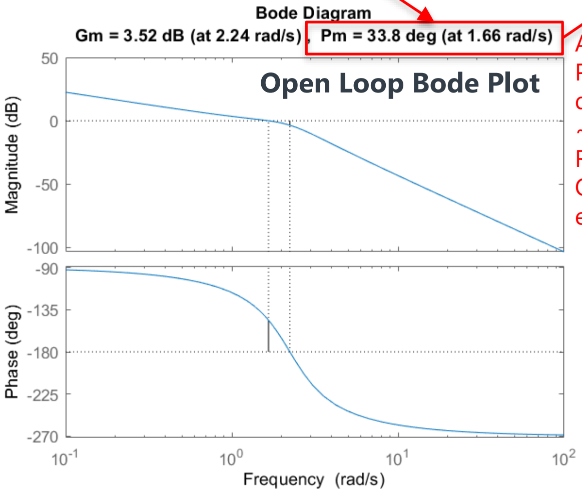
CloseLoop_Simulink_equivalent_Command.m

Design Example – Close Loop Response with Increased I Gain



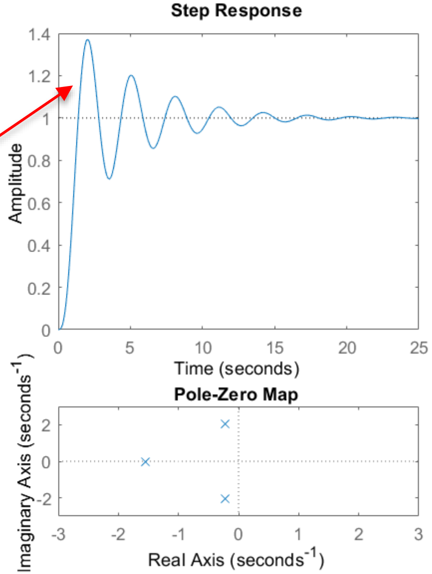
Changed from $\frac{1}{3}$ to $\frac{2}{3}$

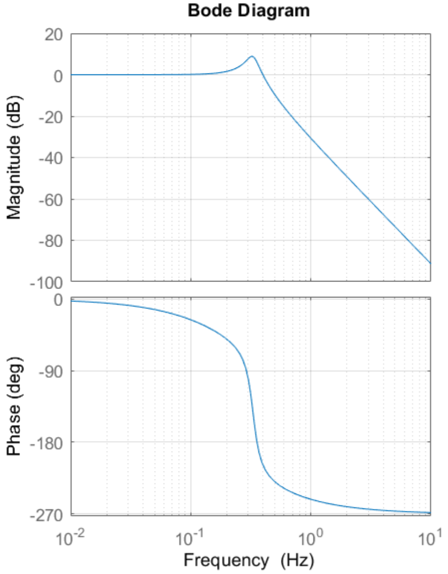
Increase Gain of Integrator, reduced phase margin



As predicted, Pm=33.8 represents damping rate is just ~0.3
Ringing and Overshoot can be expected

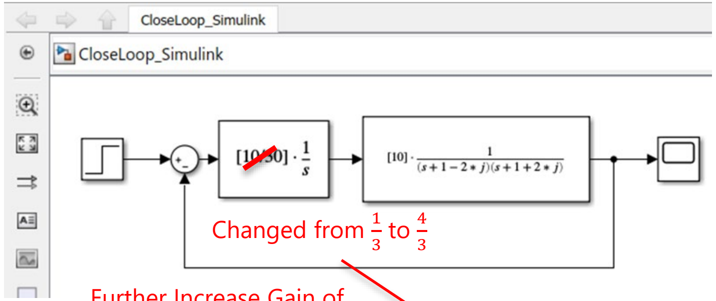
Close Loop System Response





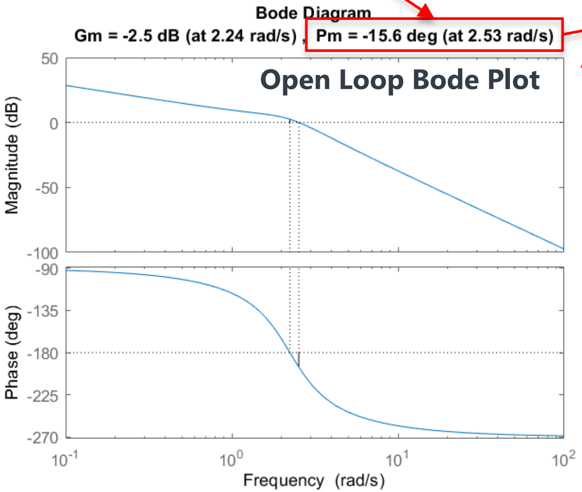
CloseLoop_Simulink_equivalent_Command.m

Design Example – Close Loop Response to Unstable



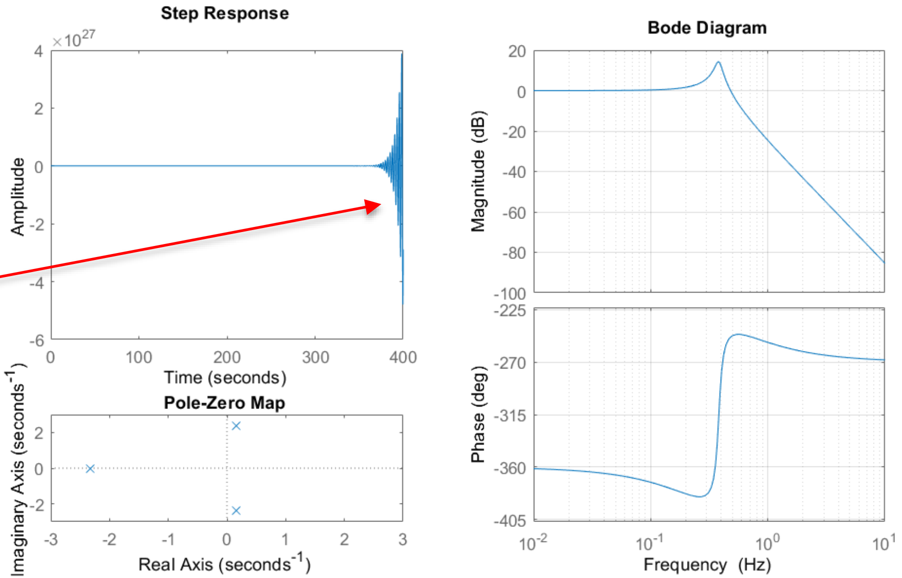
Changed from $\frac{1}{3}$ to $\frac{4}{3}$

Further Increase Gain of Integrator, Pm becomes -ve



As predicted, Pm is negative and close loop system unstable

Close Loop System Response



Appendix : System Stability for Close Loop Transfer Function

- Close Loop Transfer Function
  - $T(s) = \frac{G(s)}{1+G(s)H(s)}$
- Assume
  - $G(s) = \frac{num_G}{den_G}$ and $H(s) = \frac{num_H}{den_H}$
    - num is numerator and den is denominator
- $T(s)$ is stable
  - if all its poles are in LHP, this represent roots of $(1 + G(s)H(s)) = 0$
  - $1 + G(s)H(s) = 0$
  - $1 + \frac{num_G}{den_G} \frac{num_H}{den_H} = \frac{den_G den_H + num_G num_H}{den_G den_H} = 0$
  - $den_G den_H + num_G num_H = 0$
  - Solve above equation and to verify if all roots are in LHP

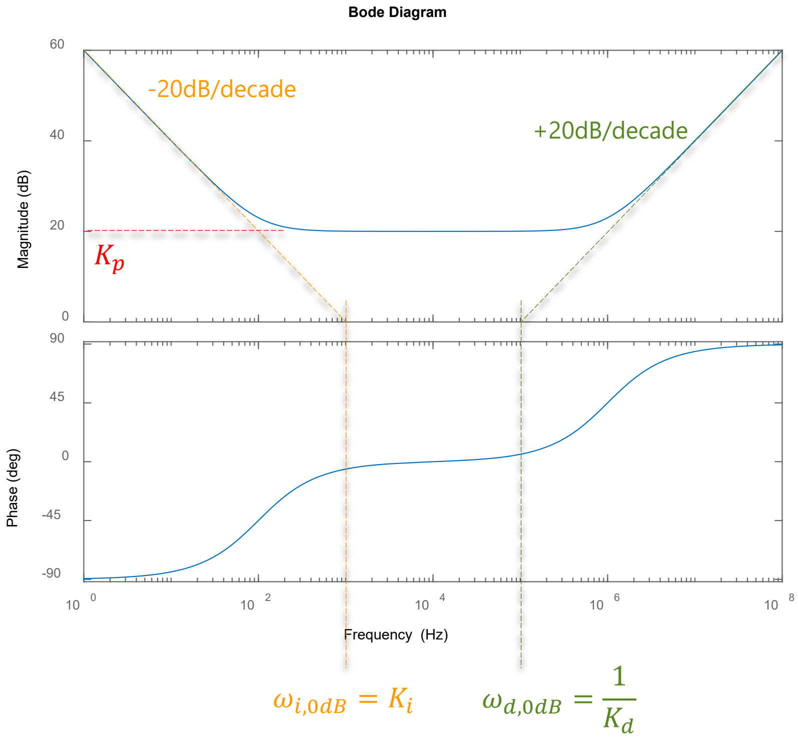
Ch4 - PID and Loop Gain Measurement

7-18-2021

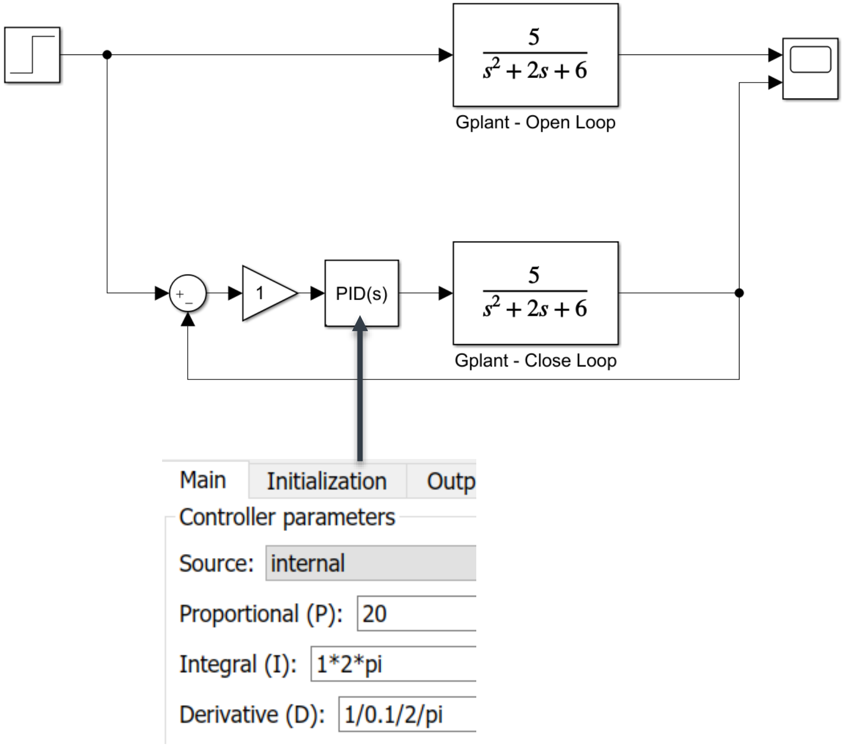
Understanding PID Controller

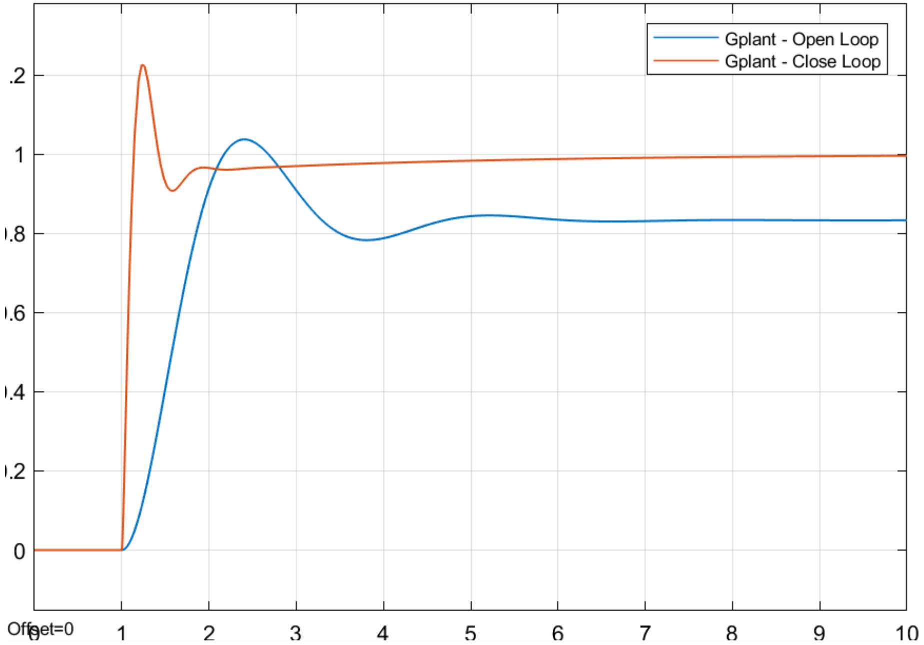
Fundamental of PID Controller

- PID Standard form
  - $u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$
- PID in Laplace form
  - $G_c(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$
- Example to explain meaning of K_p, K_i and K_d in PID format
  - $K_p = 10 = 20\log_{10}(10) = 20dB$
  - $K_i = 2\pi(10^3)$
  - $K_d = \frac{1}{2\pi(10^5)}$



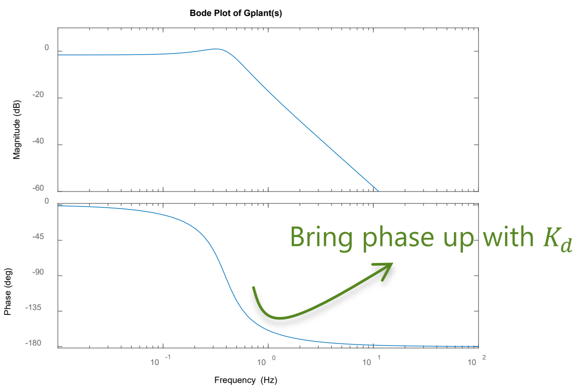
Simulink Close Loop Feedback Example with PID Controller



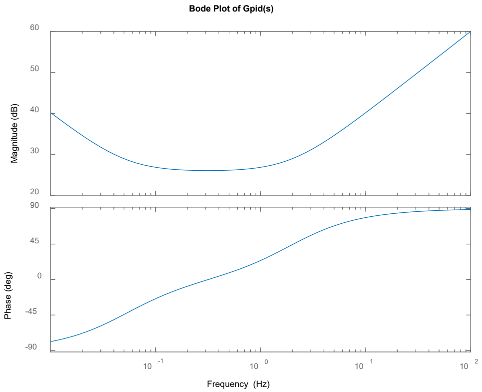


Simulink Close Loop Feedback Example with PID Controller

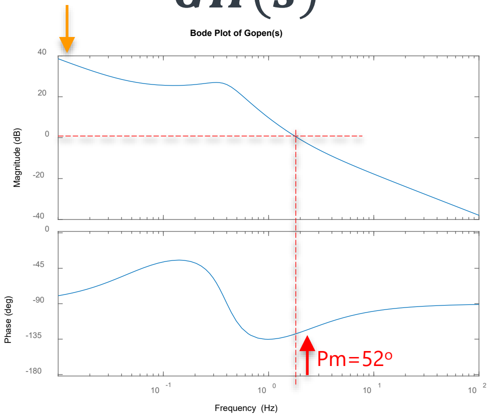
$G_{plant}(s)$



$G_{pid}(s)$



$GH(s)$

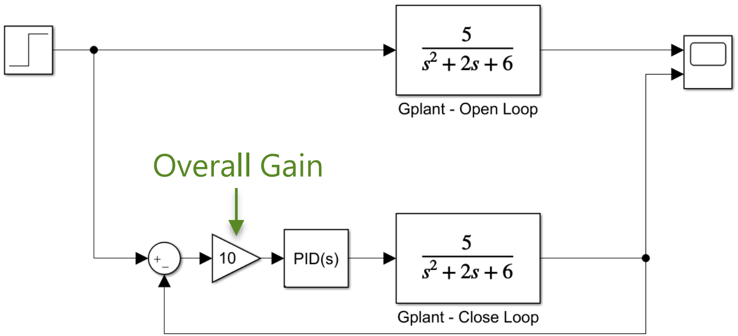


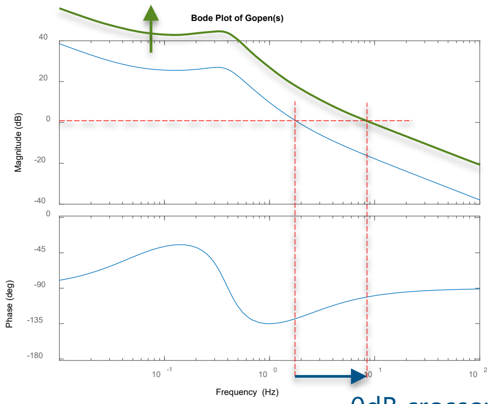
- Plant

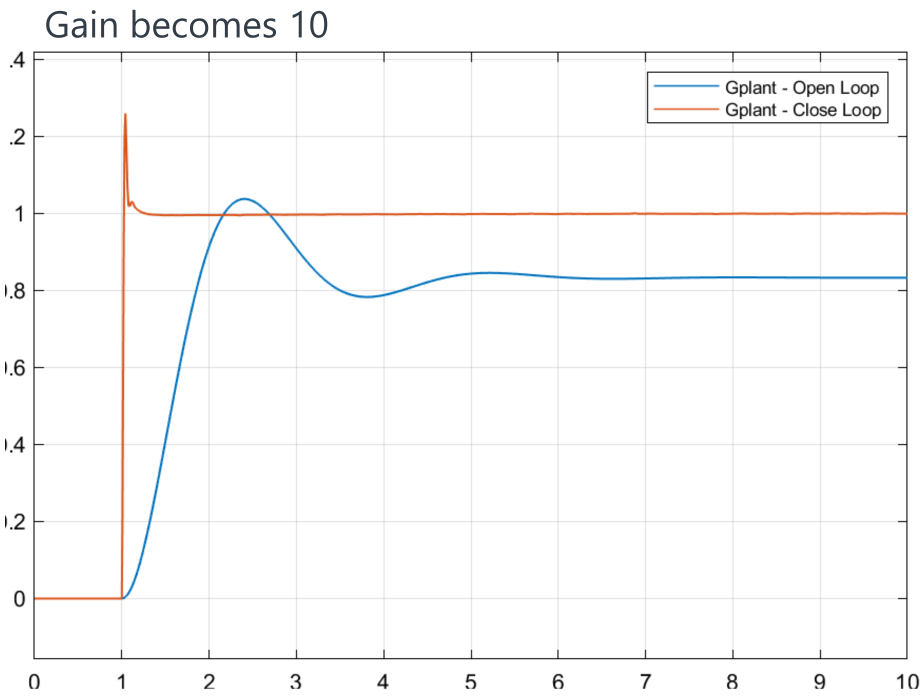
- PID Controller
  - $K_p = 20$
  - $K_i = 1 \times 2\pi$
  - $K_d = \dfrac{1}{0.1 \times 2\pi}$

- Open Loop $GH(s)$
  - $GH(s) = G_{plant}\, G_{pid}$

Simulink Close Loop Feedback Example with PID Controller

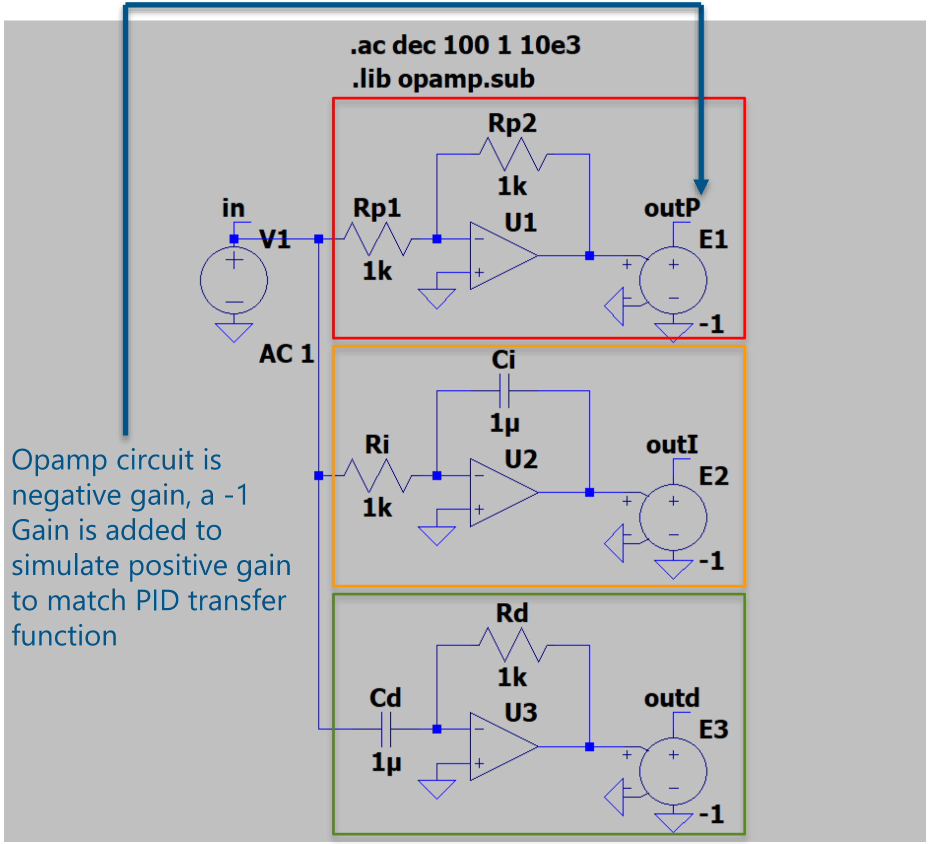






0dB crossover frequency increase, but Pm still +ve, therefore, natural frequency of close loop increase

PID with Operational Amplifier Circuit



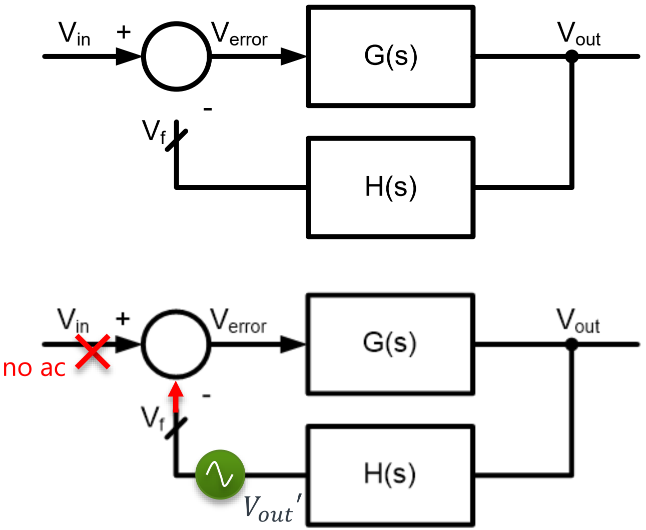
Opamp circuit is negative gain, a -1 Gain is added to simulate positive gain to match PID transfer function

- Proportional
  - $Gain_{opamp} = -\frac{Z_f}{Z_i} = -\frac{R_{p2}}{R_{p1}}$
  - $K_p = \frac{R_{p2}}{R_{p1}}$
- Integration
  - $Gain_{opamp} = -\frac{Z_f}{Z_i} = -\frac{\frac{1}{sC_i}}{R_i} = -\frac{\frac{1}{C_iR_i}}{s}$
  - $K_i = \frac{1}{C_iR_i}$
- Differentiation
  - $Gain_{opamp} = -\frac{Z_f}{Z_i} = -\frac{R_d}{\frac{1}{sC_d}} = -C_dR_ds$
  - $K_d = C_dR_d$
- In this example
  - $\omega_{i,0dB} = 2\pi f_{i,0dB} = \frac{1}{C_iR_i} \rightarrow f_{i,0dB} = \frac{1}{2\pi C_iR_i} = 159\text{Hz}$
  - $\omega_{d,0dB} = 2\pi f_{d,0dB} = \frac{1}{C_dR_d} \rightarrow f_{d,0dB} = \frac{1}{2\pi C_dR_d} = 159\text{Hz}$

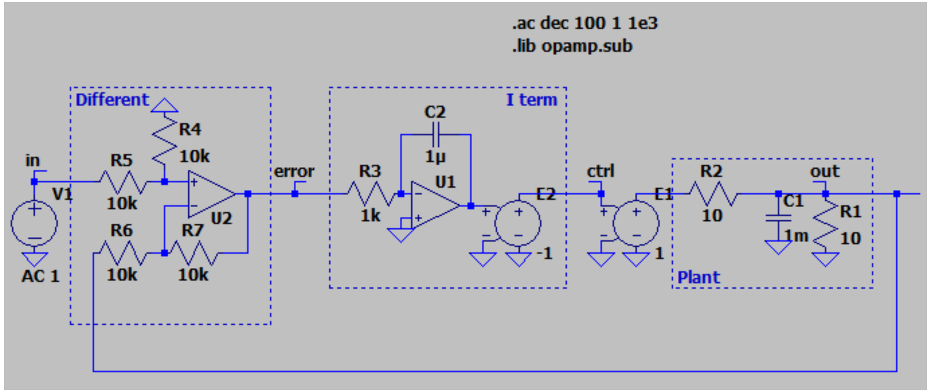
Loop Gain Measurement

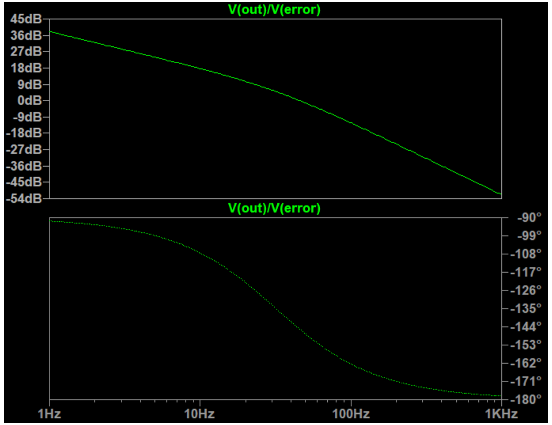
Review Open Loop Transfer Function Definition

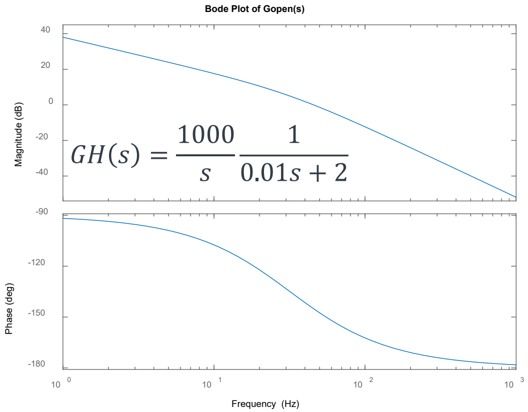
- Open Loop Transfer Function Definition
  - It is defined as cutting the feedback path as
  - $GH(s) = G(s)H(s) = G_c(s)G_{plant}(s)H(s)$
- When V_f is break from the loop and AC test signal is from V_{in}
  - $GH(s) = \frac{\tilde{v}_f}{\tilde{v}_{in}} = \frac{\tilde{v}_f}{\tilde{v}_{error}}$
- If V_{in} is DC only and inject an AC to feedback path as test signal
  - $\tilde{v}_{error} = -\tilde{v}_f$
  - $GH(s) = \frac{\tilde{v}_{out'}}{\tilde{v}_{error}} = -\frac{\tilde{v}_{out'}}{\tilde{v}_f}$
  - If $H(s) = 1$, $V_{out} = V_{out}'$
    - $GH(s) = G_c(s)G_{plant}(s) = \frac{\tilde{v}_{out}}{\tilde{v}_{error}} = -\frac{\tilde{v}_{out}}{\tilde{v}_f}$



A Simple Close-Loop System







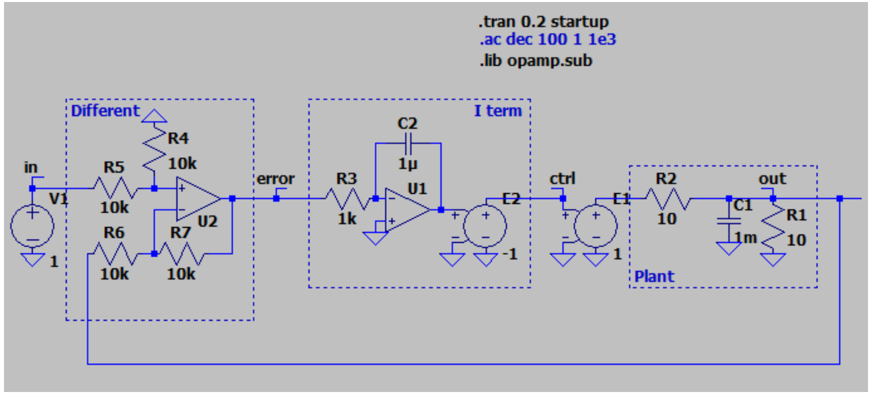
This is to simulate a simple close-loop system with only an Integrator as compensator.

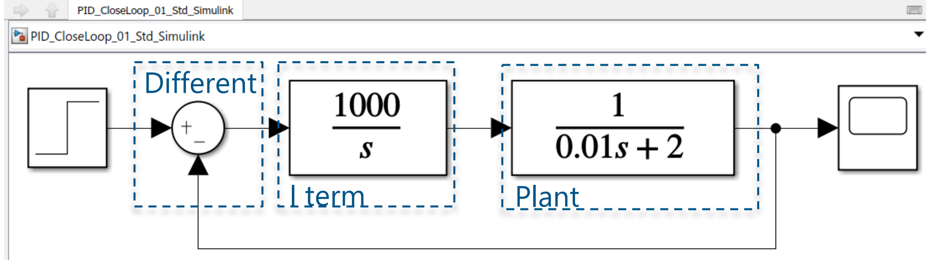
Open Loop Transfer Function is $\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{\tilde{v}_{out}}{\tilde{v}_{error}}$ if feedback is open, which is equivalent to AC sweep from v_{in} and to plot $\frac{\tilde{v}_{out}}{\tilde{v}_{error}}$

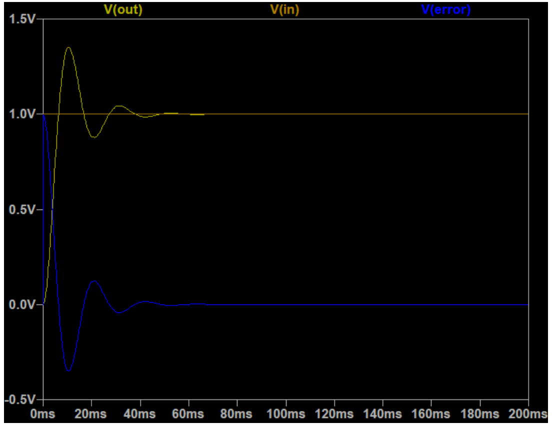
For Transfer Function:

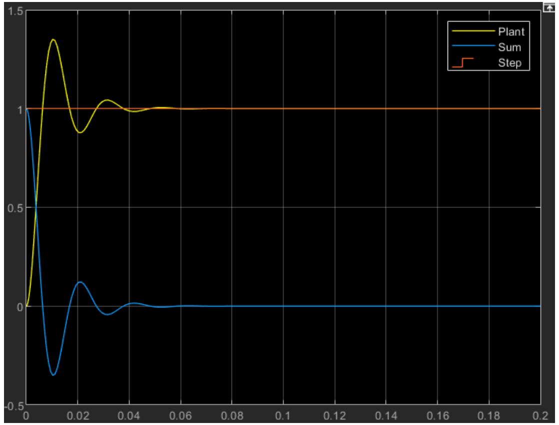
$$G_c(s) = \frac{1000}{s} \text{ and } G_p(s) = \frac{\frac{1}{\frac{1}{\frac{1}{0.001s}}+\frac{1}{10}}}{\frac{1}{\frac{1}{\frac{1}{0.001s}}+\frac{1}{10}}+10} = \frac{\frac{1}{0.001s+0.1}}{\frac{1+10(0.001s+0.1)}{0.001s+0.1}} = \frac{1}{0.01s+2}$$

Circuit and Simulink Relationship

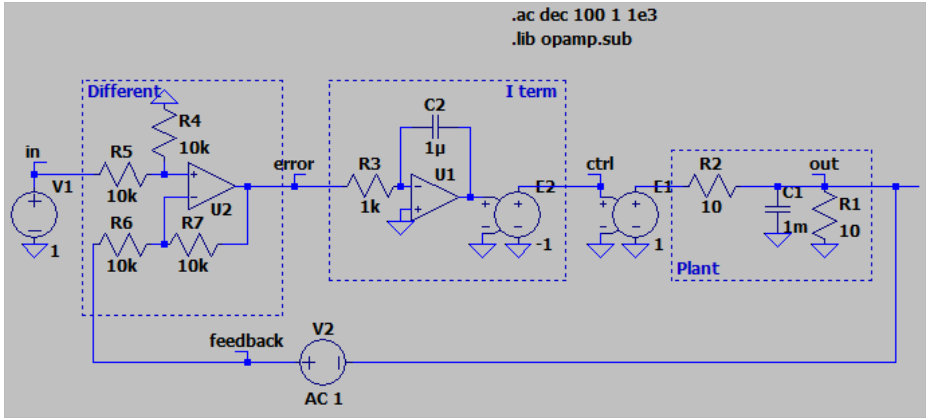


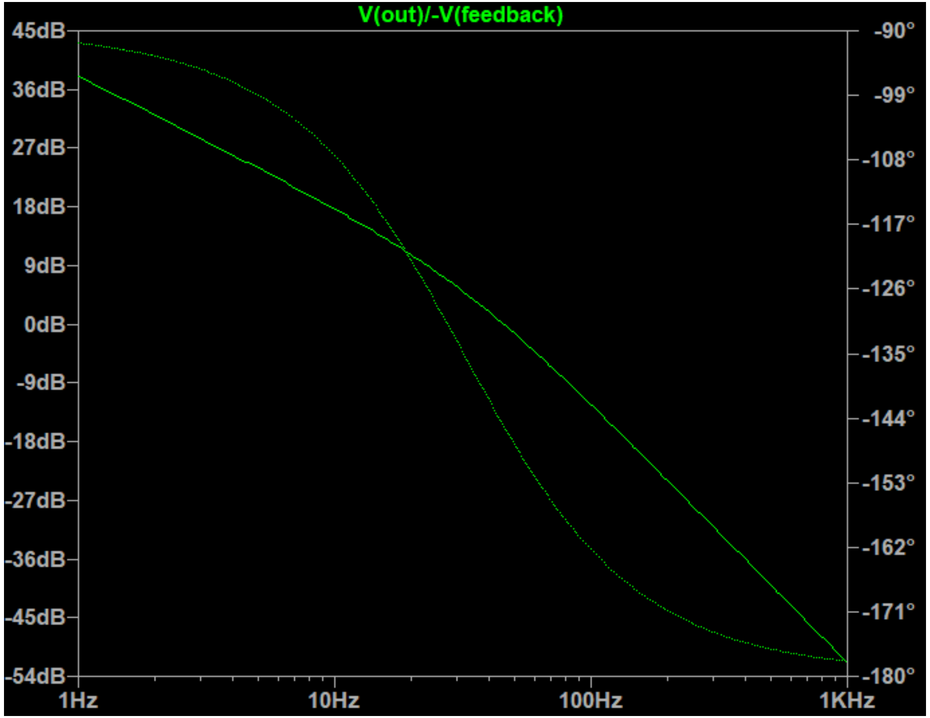






Loop Gain Measurement Injection Technique (Idea Explanation)





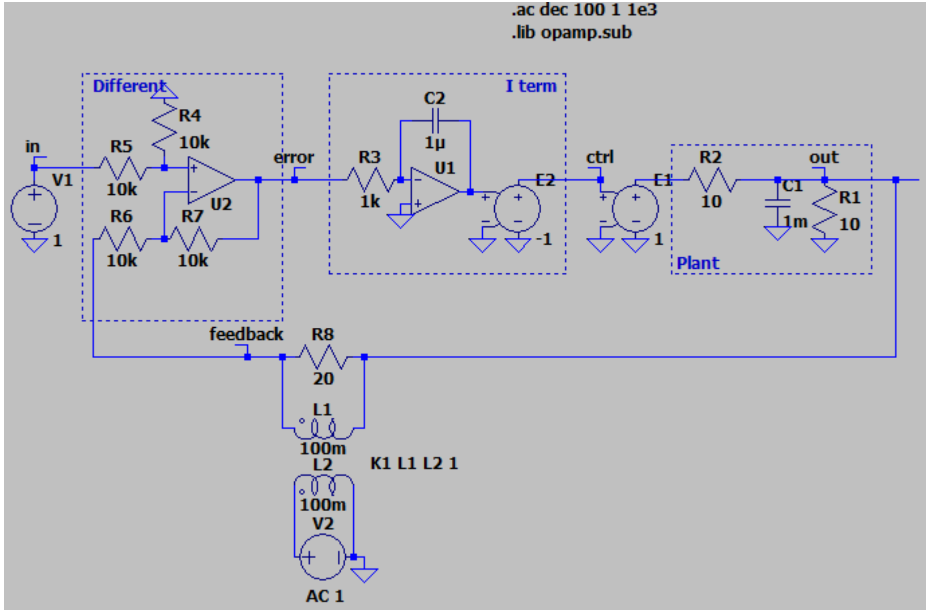
Idea to measure open loop response without breaking close loop

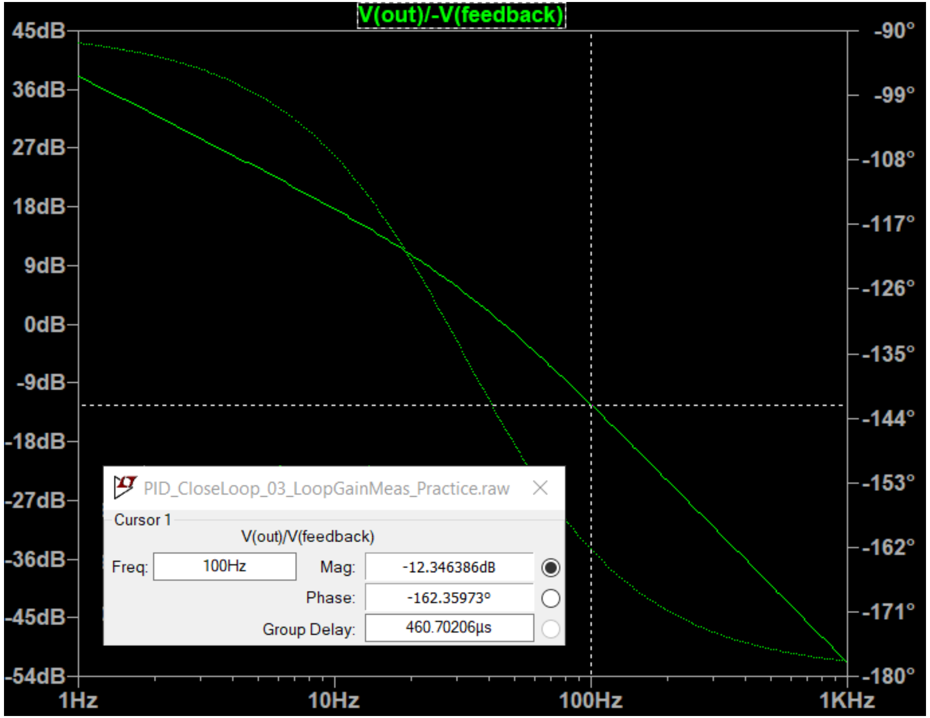
Add AC source between Vout and feedback of error amplifier

When Vin is DC signal, $\tilde{v}_{error} = -\tilde{v}_{feedback}$, therefore, cut the loop and add ac distribute, can superposition a small signal ac to feedback path to measure open loop transfer function

Sweep ac source and measure $GH(s) = -\frac{\tilde{v}_{out}}{\tilde{v}_{feedback}}$

Loop Gain Measurement Injection Technique (Practical)

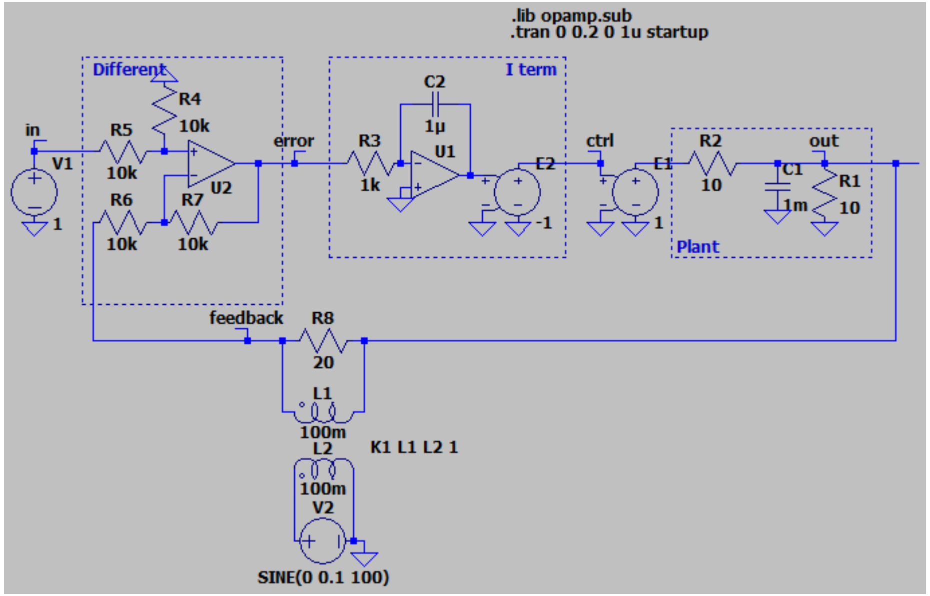


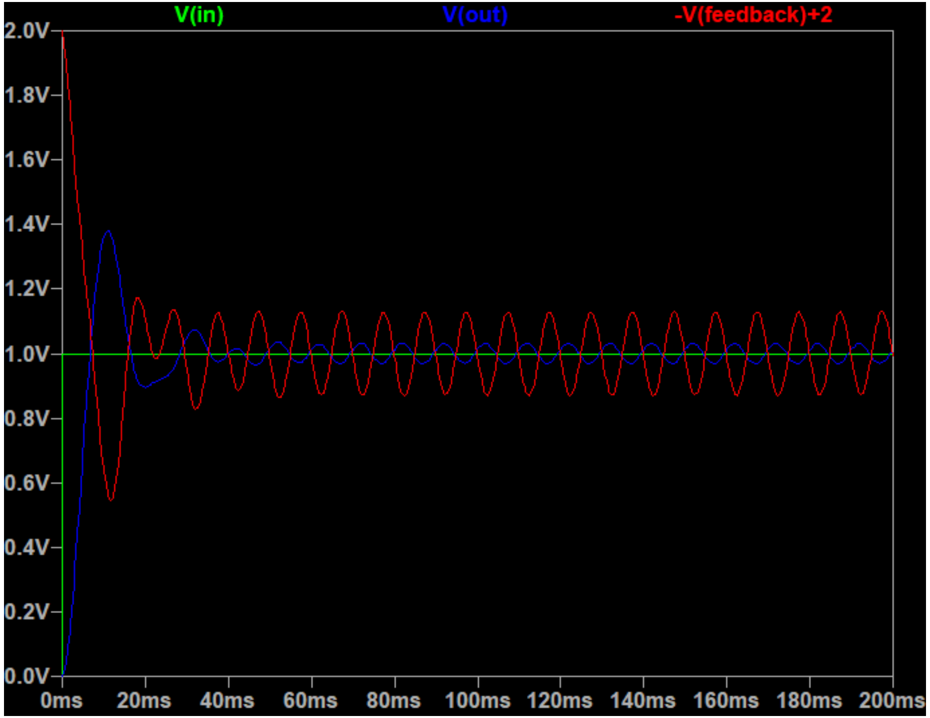


Injected ac signal need to be isolated as AC source is always GND in one end. An isolated transformer is required to isolate ac signal injection in practice.

A small resistor is added (for DC feedback path to close loop), it should be low as compare to resistance in feedback compensation network

Loop Gain Measurement Injection Technique (@100Hz)

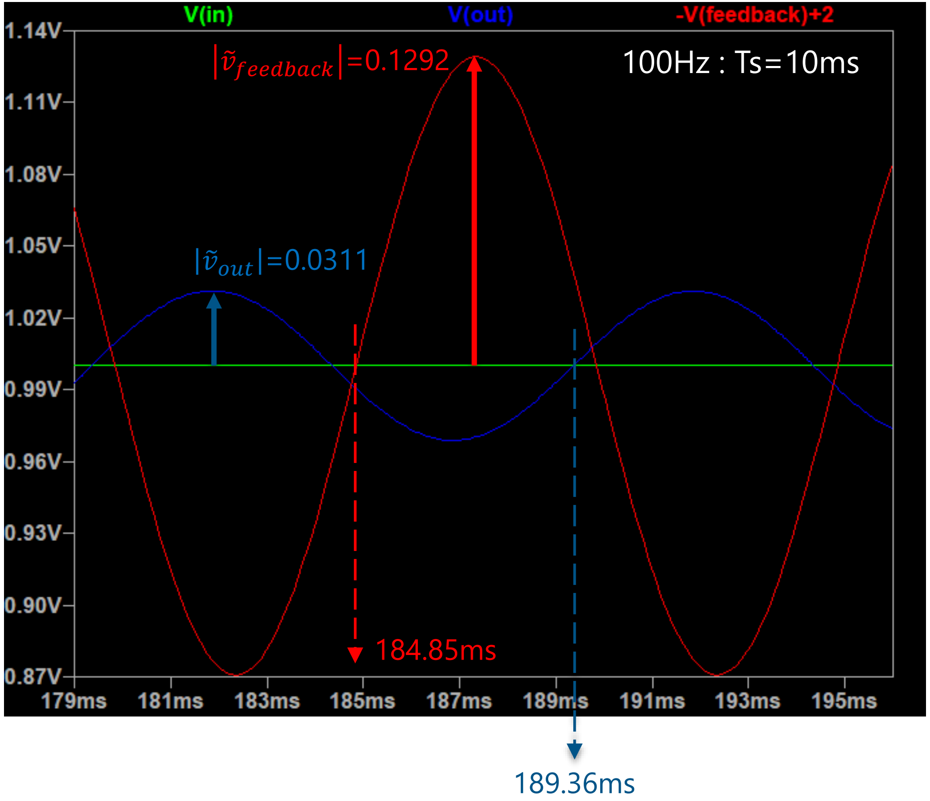




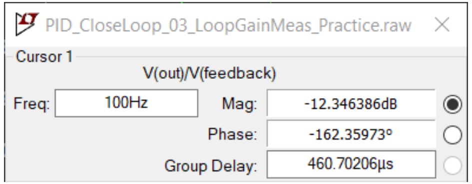
This example show a 100Hz small signal is injected into the loop
Simulate a transient response with v_{in} as a step input

$-\frac{\tilde{v}_{out}}{\tilde{v}_{feedback}}$: Gain = $\frac{|\tilde{v}_{out}|}{|\tilde{v}_{feedback}|}$ and Phase = $\frac{\angle\tilde{v}_{out}}{\angle-\tilde{v}_{feedback}} = \angle\tilde{v}_{out} - \angle(-\tilde{v}_{feedback})$

Loop Gain Measurement Injection Technique (@100Hz)



- Gain and Phase @ 100Hz
- Gain $= \dfrac{|\tilde{v}_{out}|}{|\tilde{v}_{feedback}|}$
  - Gain $= \dfrac{0.0311}{0.1292} = 0.2407$
  - Gain $= 20\log_{10}(0.2407) = -12.37\text{dB}$
- Phase $= \angle\tilde{v}_{out} - \angle(-\tilde{v}_{feedback})$
  - Assume $\angle(-\tilde{v}_{feedback})$as 0° reference
  - Phase $= \dfrac{189.36ms - 184.85ms}{10ms} \times 360° - 0°$
  - Phase $= -162.36°$

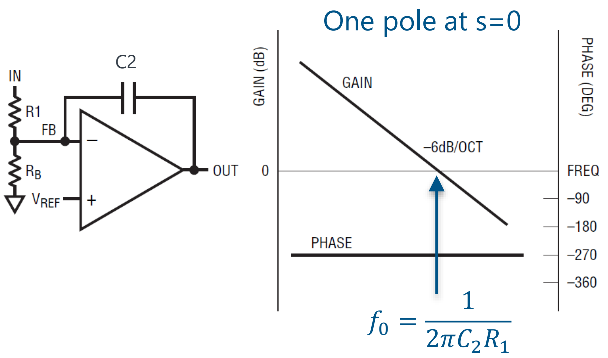


Type I to III Compensator in Power Electronics

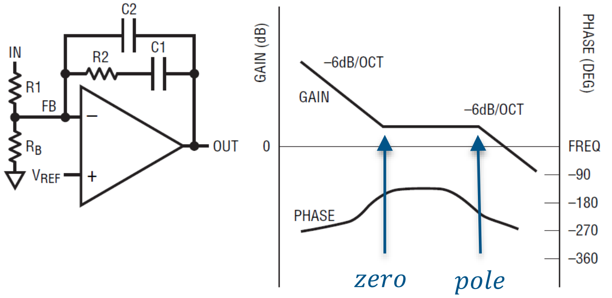
Kelvin Leung, Sr. Principal Eng
7-17-2021

Type I to III Compensator Overview

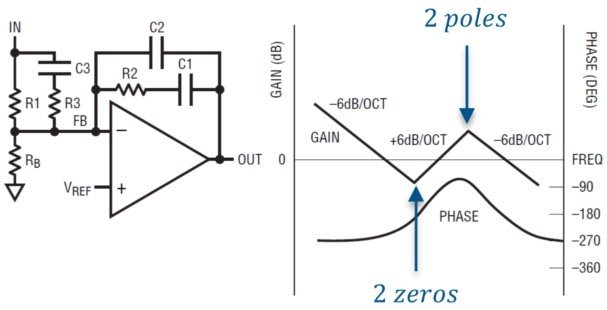
Type I



Type II

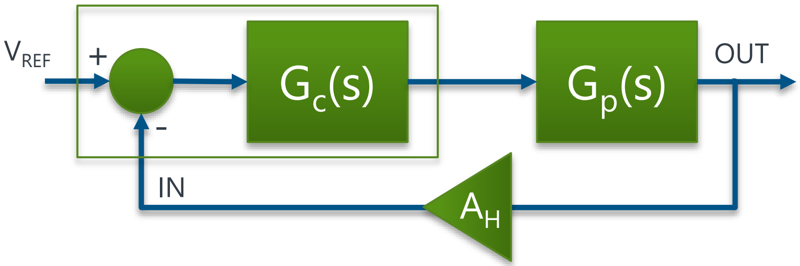


Type III

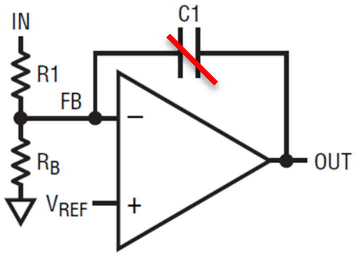


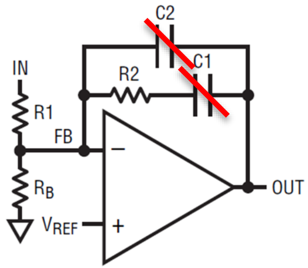
Type I to III : DC Steady State Condition

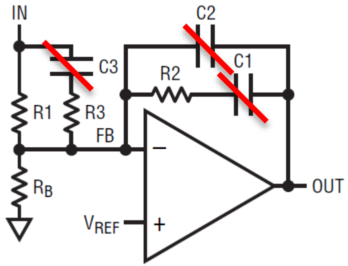
- Opamp basic formula : $v_n = v_p$, As $V_{FB} = V_{REF}$
  - $V_{REF} = V_{FB} = \frac{R_B}{R_1+R_B} V_{in}$
- Structure of Type I to III compensator includes different amplifier and $G_c(s)$



- $V_{in} = A_H V_{out} \rightarrow V_{REF} = \frac{R_B}{R_1+R_B} A_H V_{out}$
  - To calculate $V_{out} = A_H \left(\frac{R_1}{R_B} + 1\right) V_{REF}$
  - To calculate $R_B = \frac{R_1}{\frac{V_{out}}{A_H V_{REF}} - 1}$

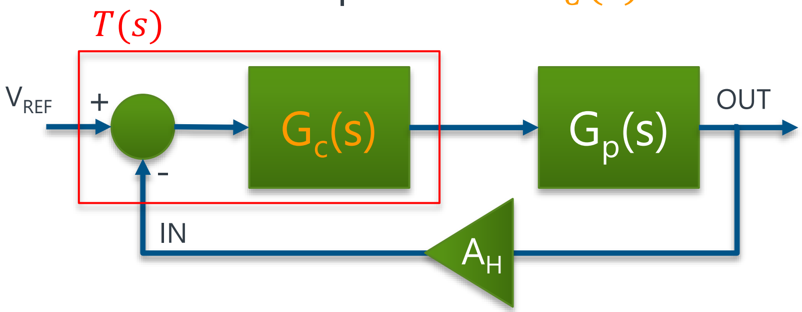


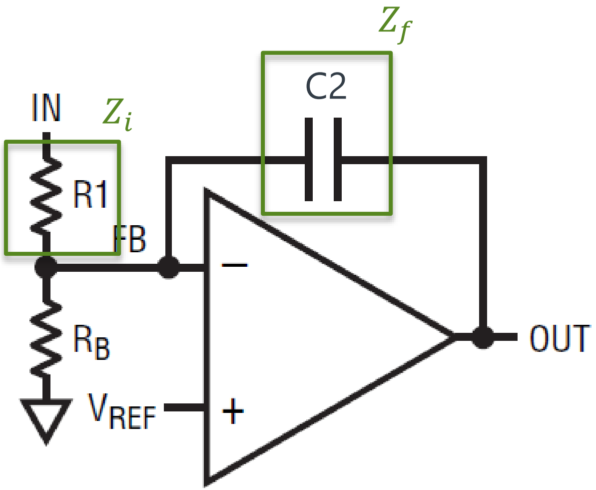




Type I Compensator Using Opamp

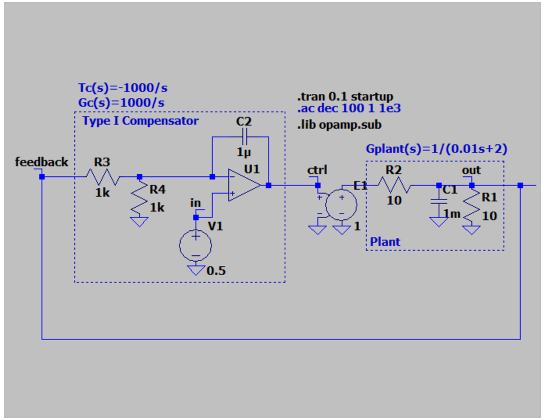
- Transfer Function
  - Error amplifier in small signal, FB=V_{REF} , R_B is ignore
  - $T(s) = -\frac{Z_f}{Z_i} = -\frac{\frac{1}{sC_2}}{R_1} = -\frac{1}{sC_2R_1}$
- Refer to control block diagram, Type I compensator $T(s)$ include different amplifier and $G_C(s)$

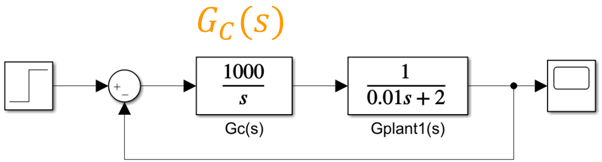


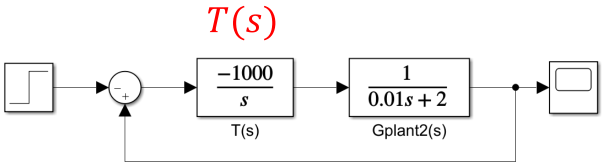


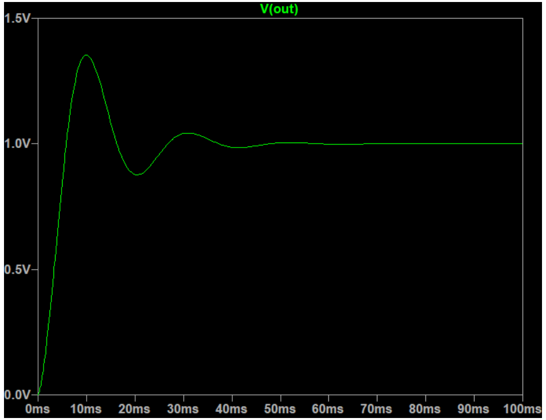
- Therefore $T(s) = -G_C(s) \rightarrow G(s) = \frac{1}{sC_2R_1}$
- Bode
  - $f_0 = \frac{1}{2\pi C_2R_1}$

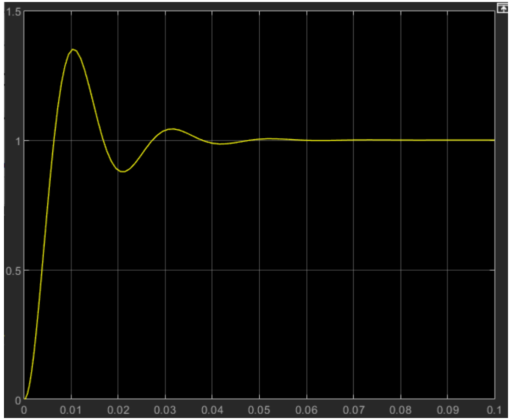
Type I Compensator - $T(s)$ and $G_c(s)$ Relationship

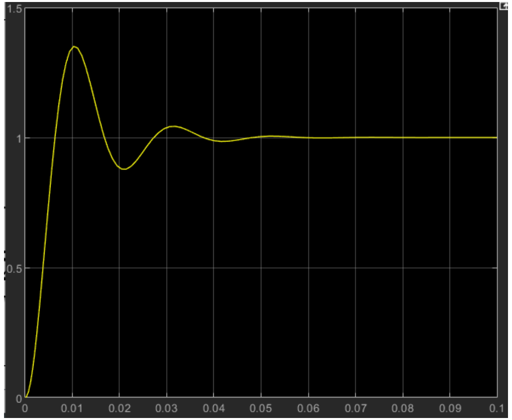




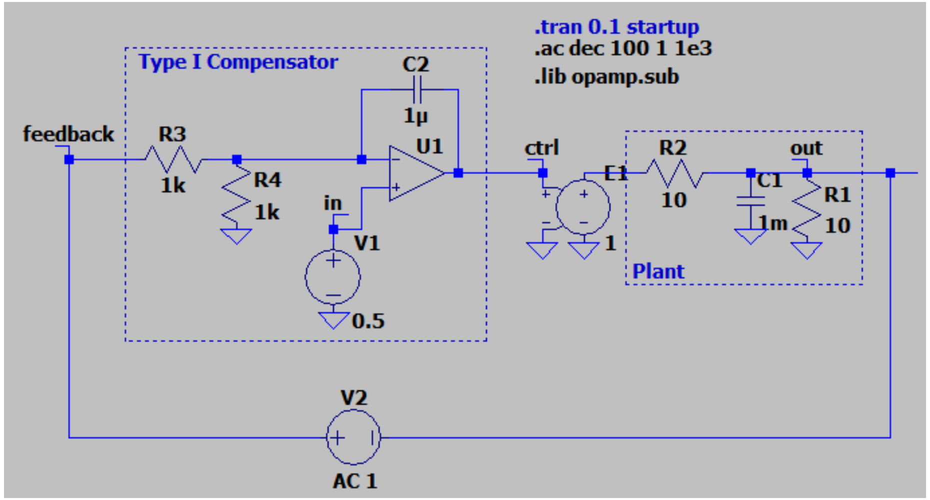


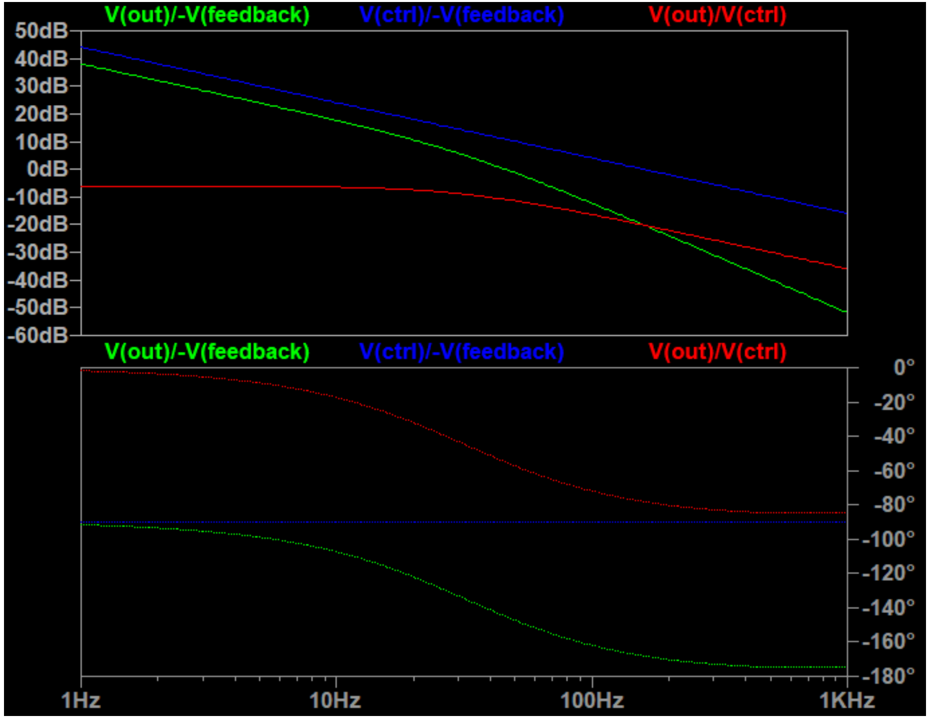






Loop Gain Measurement Injection Technique

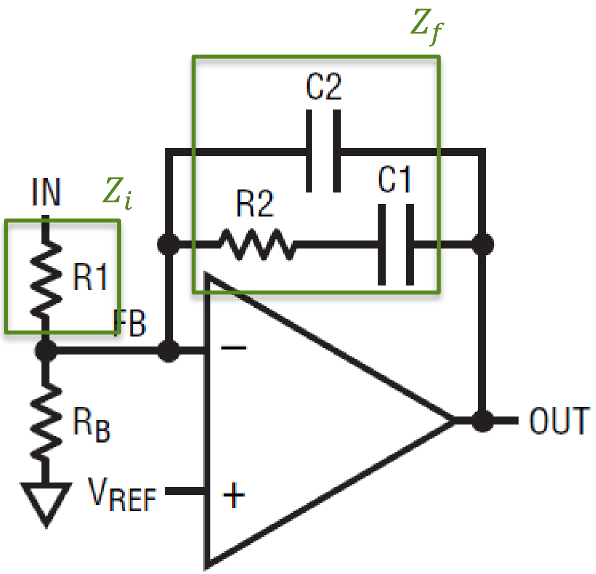




Add AC source between Vout and feedback of error amplifier
When Vin is DC signal,
Sweep ac source and measure $GH(s) = -\frac{\tilde{v}_{out}}{\tilde{v}_{feedback}}$

Type II Compensator Using Opamp

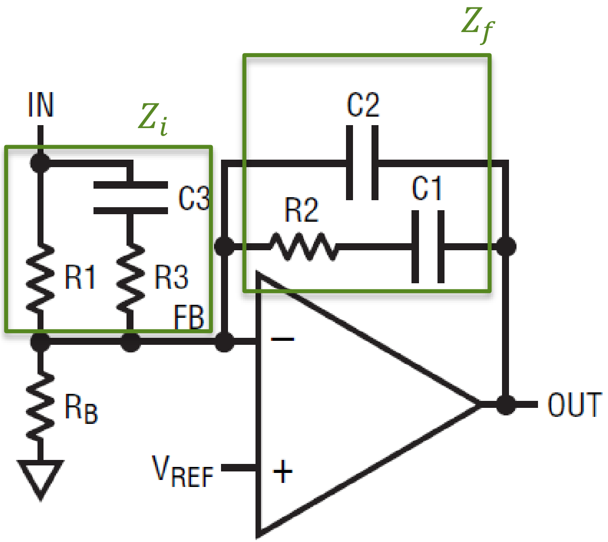
- Transfer Function
  - $Z_i = R_1$
  - $Z_f = \frac{1}{sC_2} \parallel R_2 + \frac{1}{sC_1} = \frac{1}{\frac{1}{\frac{1}{sC_2}}+\frac{1}{R_2+\frac{1}{sC_1}}} = \frac{1}{sC_2+\frac{sC_1}{sC_1R_2+1}} = \frac{1}{\frac{sC_2(sC_1R_2+1)+sC_1}{sC_1R_2+1}} = \frac{sC_1R_2+1}{s(s\,C_1C_2R_2+(C_1+C_2))}$
  - $T(s) = -\frac{Z_f}{Z_i} = -\frac{\frac{sC_1R_2+1}{s(s\,C_1C_2R_2+(C_1+C_2))}}{R_1} = -\frac{sC_1R_2+1}{s\,(s\,C_1C_2R_1R_2+(C_1+C_2)R_1)} = -\frac{C_1R_2\left(s+\frac{1}{C_1R_2}\right)}{C_1C_2R_1R_2\,s\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)} = -\frac{1}{sC_2R_1}\frac{\left(s+\frac{1}{C_1R_2}\right)}{\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)}$
  - $G_c(s) = \frac{1}{sC_2R_1}\frac{\left(s+\frac{1}{C_1R_2}\right)}{\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)}$
- Bode
  - $f_0 = \frac{1}{2\pi}\frac{1}{C_2R_1}$
  - $f_{z1} = \frac{1}{2\pi}\frac{1}{C_1R_2}$
  - $f_{p1} = \frac{1}{2\pi}\frac{C_1+C_2}{C_1C_2R_2}$



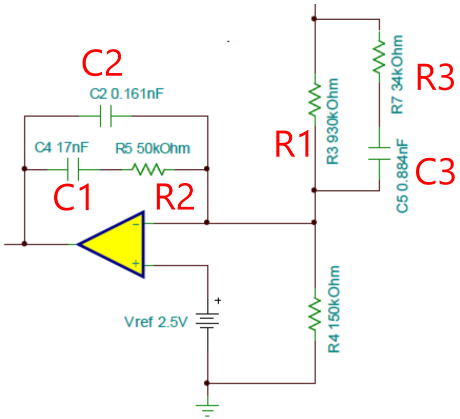
Type III Compensator Using Opamp

- Transfer Function
  - $Z_i = R_1 \parallel R_3 + \frac{1}{sC_3} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_3+\frac{1}{sC_3}}} = \frac{1}{\frac{1}{R_1}+\frac{sC_3}{sC_3R_3+1}} = \frac{1}{\frac{sC_3R_3+1+sC_3R_1}{R_1(sC_3R_3+1)}} = \frac{R_1(sC_3R_3+1)}{sC_3(R_1+R_3)+1}$
  - $Z_f = \frac{1}{sC_2} \parallel R_2 + \frac{1}{sC_1} = \frac{sC_1R_2+1}{s(s\,C_1C_2R_2+(C_1+C_2))}$
  - $T(s) = -\frac{Z_f}{Z_i} = -\frac{\frac{sC_1R_2+1}{s(sC_1C_2R_2+(C_1+C_2))}}{\frac{R_1(sC_3R_3+1)}{sC_3(R_1+R_3)+1}} = -\frac{(sC_1R_2+1)(sC_3(R_1+R_3)+1)}{R_1s(sC_1C_2R_2+(C_1+C_2))(sC_3R_3+1)} =$
    $-\frac{C_1C_3R_2(R_1+R_3)\left(s+\frac{1}{C_1R_2}\right)\left(s+\frac{1}{C_3(R_1+R_3)}\right)}{C_1C_2C_3R_1R_2R_3\,s\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)\left(s+\frac{1}{C_3R_3}\right)} = -\frac{1}{sC_2\left(\frac{R_1R_3}{R_1+R_3}\right)}\frac{\left(s+\frac{1}{C_1R_2}\right)\left(s+\frac{1}{C_3(R_1+R_3)}\right)}{\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)\left(s+\frac{1}{C_3R_3}\right)}$
  - $G_c(s) = \frac{1}{sC_2\left(\frac{R_1R_3}{R_1+R_3}\right)}\frac{\left(s+\frac{1}{C_1R_2}\right)\left(s+\frac{1}{C_3(R_1+R_3)}\right)}{\left(s+\frac{C_1+C_2}{C_1C_2R_2}\right)\left(s+\frac{1}{C_3R_3}\right)}$

- Bode
  - $f_0 = \frac{1}{2\pi}\frac{1}{C_2\left(\frac{R_1R_3}{R_1+R_3}\right)}$
  - $f_{z1} = \frac{1}{2\pi}\frac{1}{C_1R_2}$, $f_{z2} = \frac{1}{2\pi}\frac{1}{C_3(R_1+R_3)}$
  - $f_{p1} = \frac{1}{2\pi}\frac{C_1+C_2}{C_1C_2R_2}$, $f_{p2} = \frac{1}{2\pi}\frac{1}{C_3R_3}$

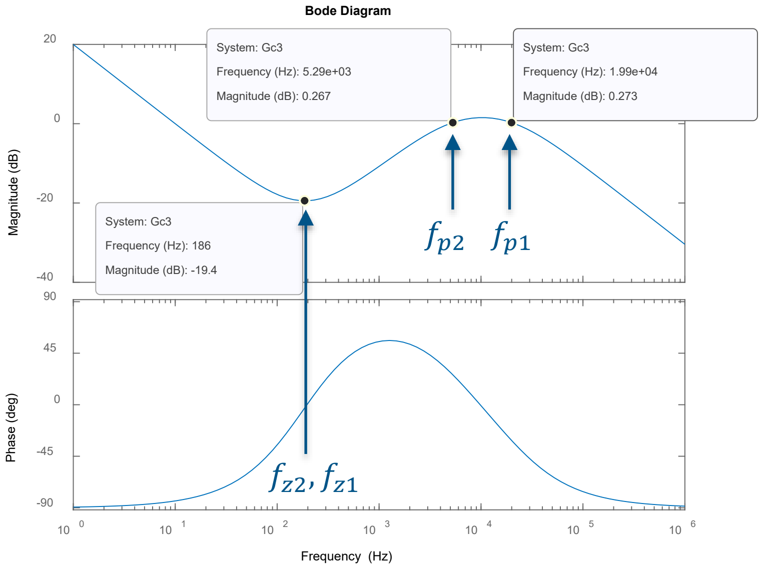


Type III Compensator in Texas Instrument SLVA633

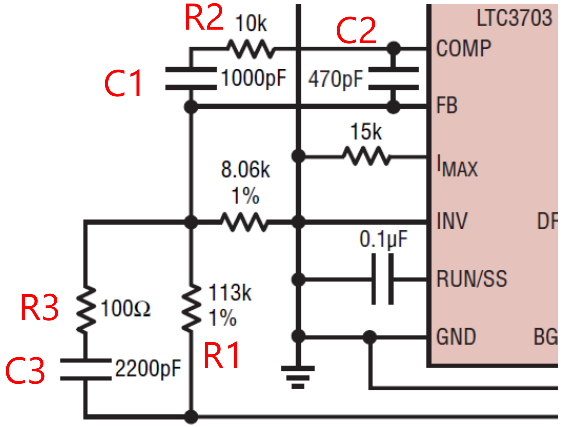


- Calculated Frequency
  - $f_0 = 30.138kHz$
  - $f_{z1} = 187.24Hz$
  - $f_{z2} = 186.76Hz$
  - $f_{p1} = 19.958kHz$
  - $f_{p2} = 5.2953kHz$

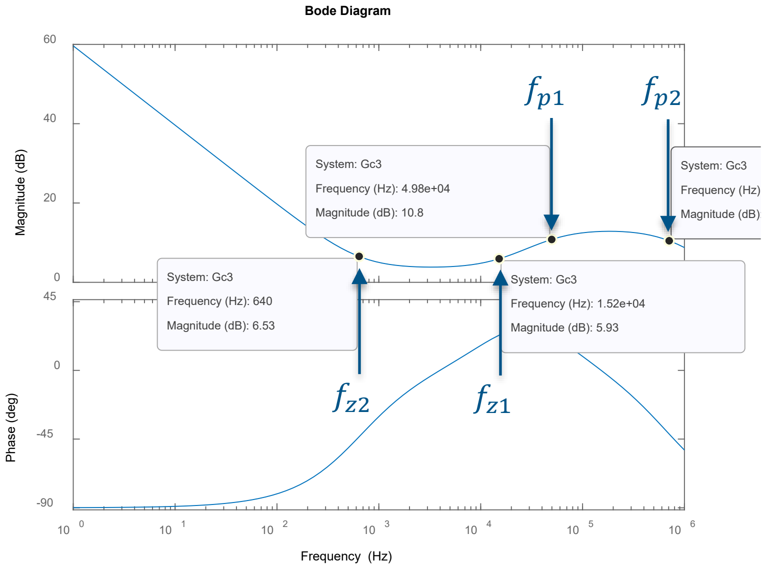
Transfer Function $G_c(S)$



Type III Compensator in LTC3703 Datasheet



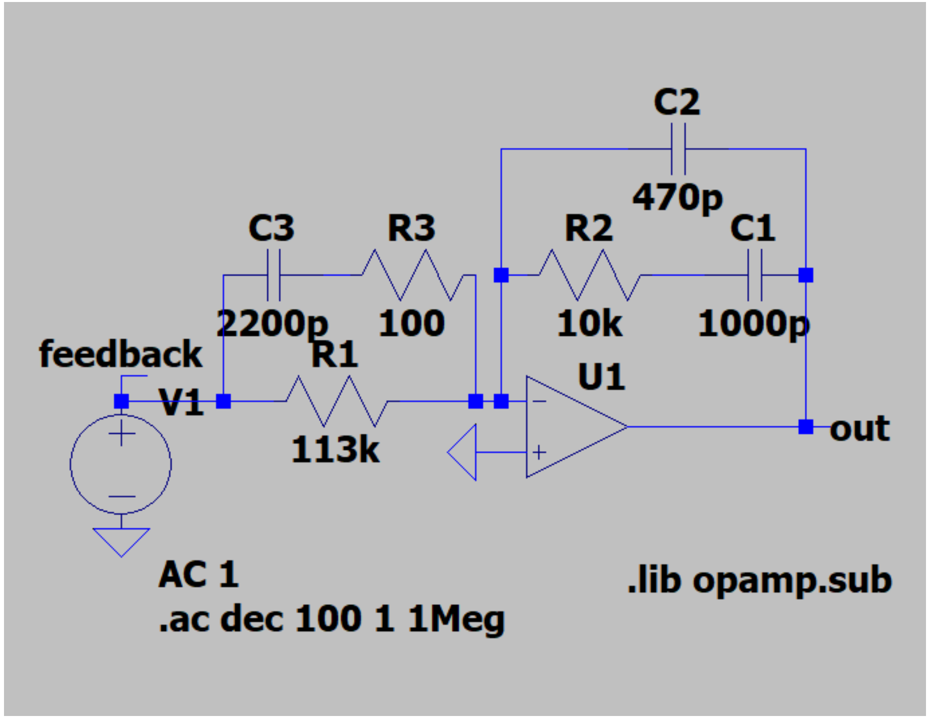
Transfer Function $G_c(S)$



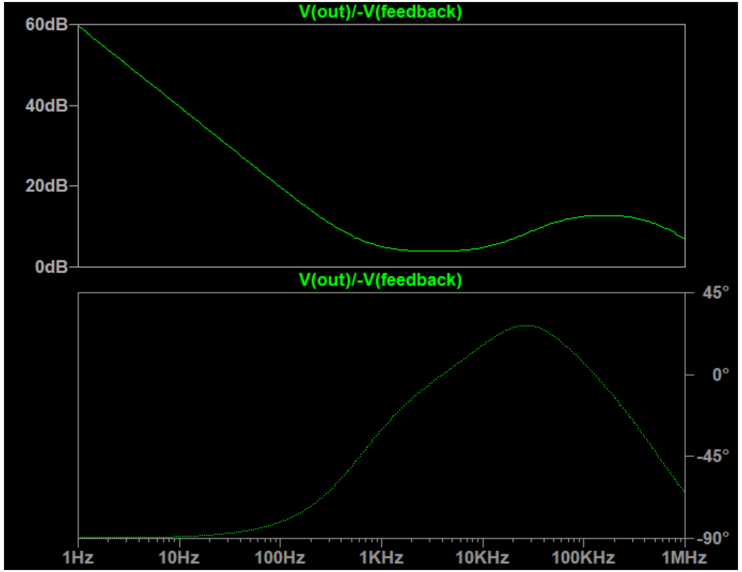
- Calculated Frequency
  - $f_0 = 3.3893MHz$
  - $f_{z1} = 15.915kHz$
  - $f_{z2} = 639Hz$
  - $f_{p1} = 49.778kHz$
  - $f_{p2} = 723.43kHz$

Type III Compensator in LTC3703 Datasheet

LTspice AC Sweep can be used to plot Bode Plot



Transfer Function $G_c(S)$



TypeIII_Compensator_Bode.asc

Calculate Type III R & C from f_{z1}, f_{z2}, f_{p1}, f_{p2}, R_1 and R_2

- User Input Parameters
  - f_{z1}, f_{z2}, f_{p1} and f_{p2}
  - R_1 and R_2

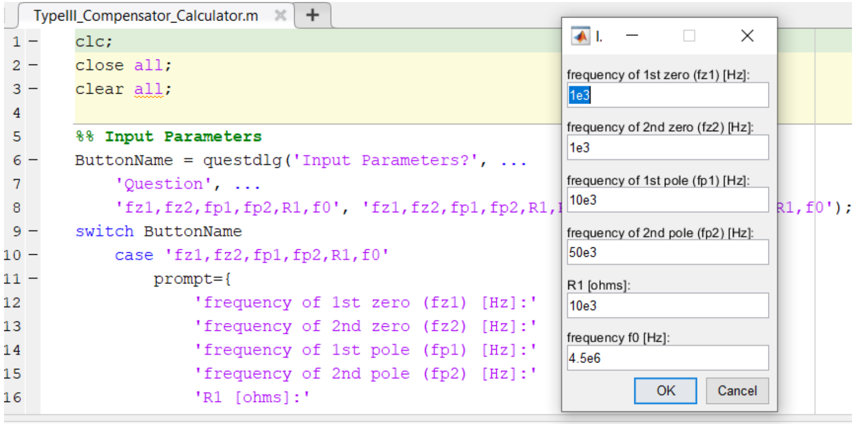
- Basic Formula for Type III Compensator
  - $f_0 = \frac{1}{2\pi}\frac{1}{C_2\left(\frac{R_1R_3}{R_1+R_3}\right)}$, $f_{z1} = \frac{1}{2\pi}\frac{1}{C_1R_2}$, $f_{z2} = \frac{1}{2\pi}\frac{1}{C_3(R_1+R_3)}$, $f_{p1} = \frac{1}{2\pi}\frac{C_1+C_2}{C_1C_2R_2}$, $f_{p2} = \frac{1}{2\pi}\frac{1}{C_3R_3}$

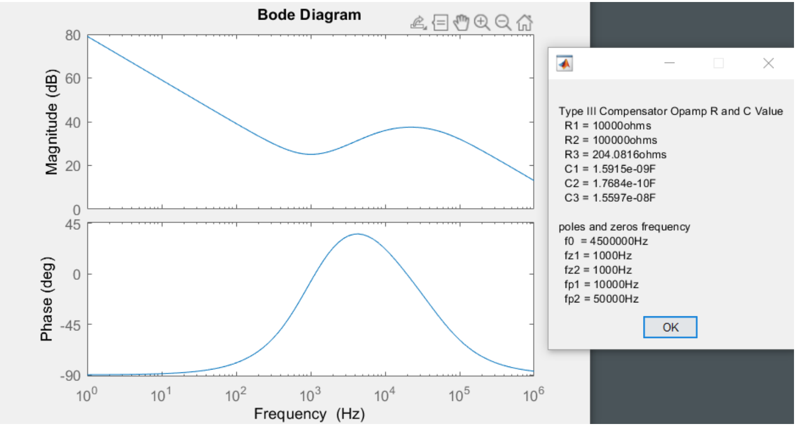
- Calculation
  - Put R_1 to calculate C_3 and R_3
    - By eqn f_{p2} : $C_3 = \frac{1}{2\pi f_{p2}R_3}$ and eqn f_{z2} : $R_1 = \frac{1}{2\pi f_{z2}C_3} - R_3$, can calculate $R_1 = \frac{1}{2\pi f_{z2}C_3} - \frac{1}{2\pi f_{p2}C_3} = \frac{1}{2\pi C_3}\left(\frac{1}{f_{z2}} - \frac{1}{f_{p2}}\right)$
    - $C_3 = \frac{1}{2\pi R_1}\left(\frac{1}{f_{z2}} - \frac{1}{f_{p2}}\right)$
    - $R_3 = \frac{1}{2\pi f_{p2}C_3}$
  - Put R_2 to calculate C_1 and C_2
    - By eqn f_{z1} : $C_1 = \frac{1}{2\pi f_{z1}R_2}$
    - By eqn f_{p1} : $C_2 = \frac{1}{R_2\left(2\pi f_{p1} - \frac{1}{C_1R_2}\right)}$

Calculate Type III R & C from $f_{z1}, f_{z2}, f_{p1}, f_{p2}, f_0$ and R_1

- User Input Parameters
  - f_{z1}, f_{z2}, f_{p1} and f_{p2}
  - f_0 : higher this value, overall gain becomes higher
- Basic Formula for Type III Compensator
  - $f_0 = \frac{1}{2\pi}\frac{1}{C_2\left(\frac{R_1R_3}{R_1+R_3}\right)}$, $f_{z1} = \frac{1}{2\pi}\frac{1}{C_1R_2}$, $f_{z2} = \frac{1}{2\pi}\frac{1}{C_3(R_1+R_3)}$, $f_{p1} = \frac{1}{2\pi}\frac{C_1+C_2}{C_1C_2R_2}$, $f_{p2} = \frac{1}{2\pi}\frac{1}{C_3R_3}$
- Calculation
  - Put R_1 to calculate C_3 and R_3
    - By eqn f_{p2} : $C_3 = \frac{1}{2\pi f_{p2}R_3}$ and eqn f_{z2} : $R_1 = \frac{1}{2\pi f_{z2}C_3} - R_3$, can calculate $R_1 = \frac{1}{2\pi f_{z2}C_3} - \frac{1}{2\pi f_{p2}C_3} = \frac{1}{2\pi C_3}\left(\frac{1}{f_{z2}} - \frac{1}{f_{p2}}\right)$
    - → $C_3 = \frac{1}{2\pi R_1}\left(\frac{1}{f_{z2}} - \frac{1}{f_{p2}}\right)$
    - By eqn f_{p2} : $R_3 = \frac{1}{2\pi f_{p2}C_3}$
  - Put f_0 to calculate C_1 and R_2
    - By eqn f_0 : $C_2 = \frac{1}{2\pi}\frac{1}{f_0\left(\frac{R_1R_3}{R_1+R_3}\right)}$
    - By eqn f_{p1} : $f_{p1} = \frac{1}{2\pi}\frac{C_1+C_2}{C_1C_2\frac{1}{2\pi f_{z1}C_1}}$, can calculate $\frac{f_{p1}}{f_{z1}} = \frac{C_1+C_2}{C_2}$
    - → $C_1 = \left(\frac{f_{p1}}{f_{z1}} - 1\right) C_2$
    - By eqn f_{z1} : $R_2 = \frac{1}{2\pi f_{z1}C_1}$

TypeIII_Compensator_Calculator.m

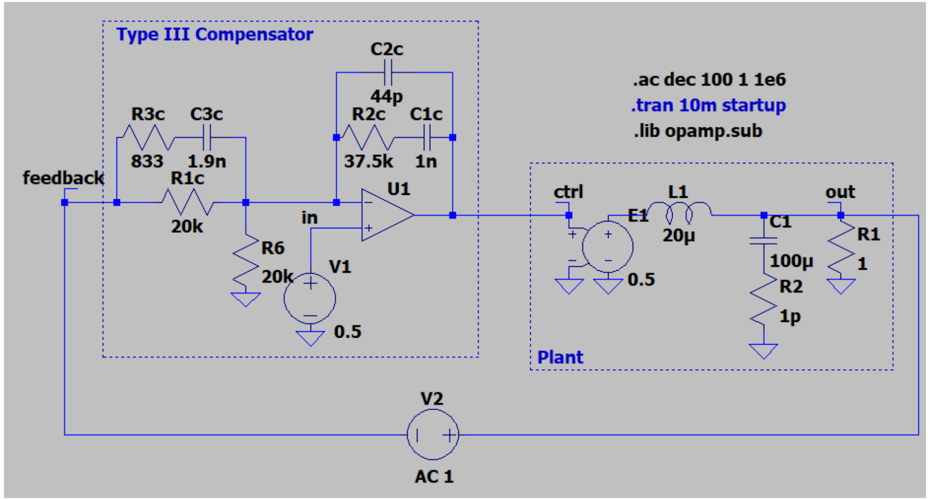


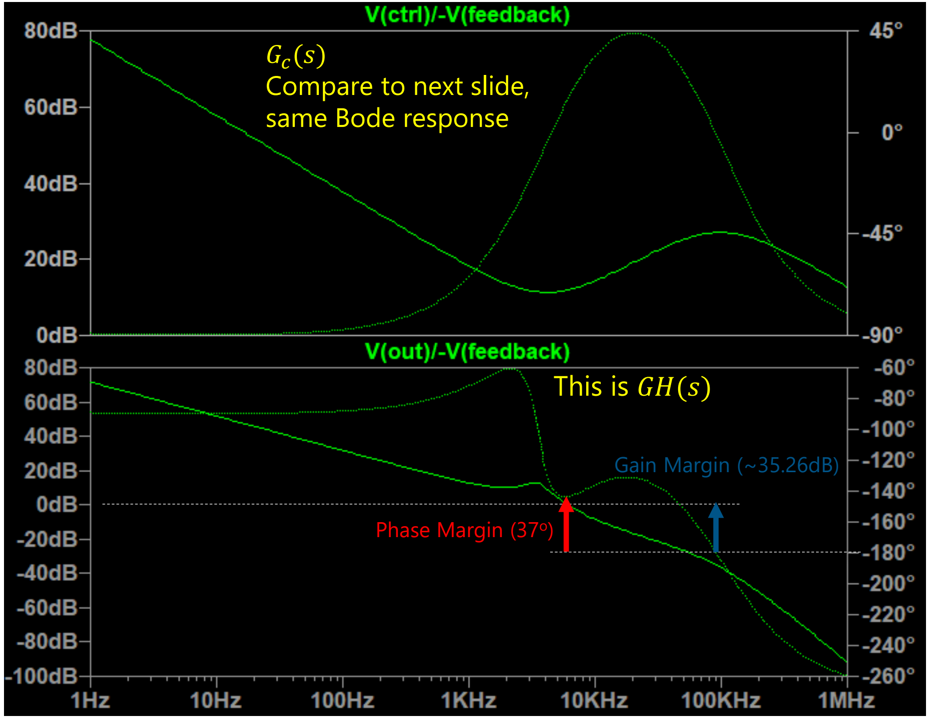


- A matlab script TypeIII_Compensator_Calculator.m is written to help engineer compute resistors and capacitors value to achieved desired type III compensator bode response

Loop Gain Measurement with Type III Compensator

Loop Gain Measurement Injection Technique

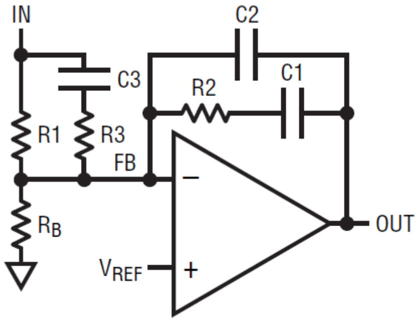


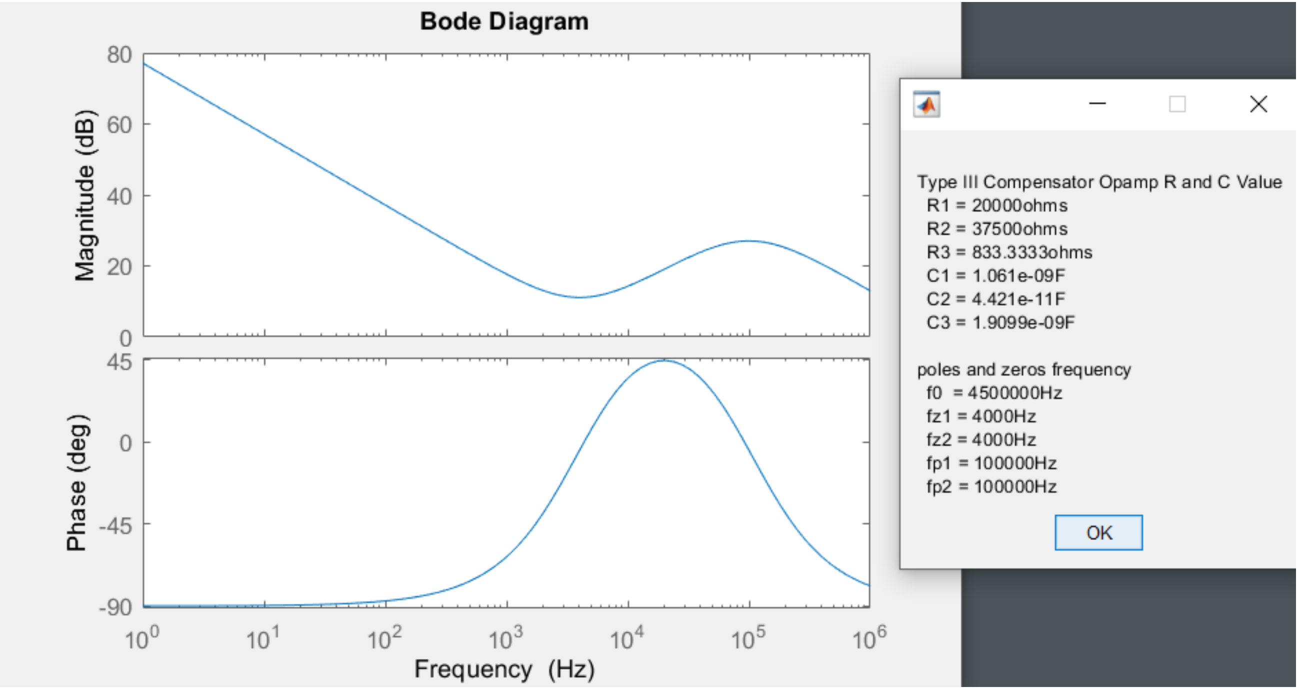


Compare to chapter 4, an ac small signal test signal can be used to identify system transfer function

$$GH(s) = -\frac{\tilde{v}_{out}}{\tilde{v}_{feedback}}$$

Type III Compensator Calculated by Matlab





Transient Response Simulation